Insider Investor and Information*

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Abstract

Relationship financing of innovative projects, as is common in bank lending and venture capital, features incumbent financiers' observing interim information before deciding on continued financing. Entrepreneurs' endogenous experimentation reduces the insider investor's information monopoly rent, but the moral hazard of information production in such persuasion games reduces the incentives for initial investment. Insiders' independent information and competition mitigate the hold-up problem and, consistent with empirical observations, have non-monotone effects on relationship formation. Because experimentation only alters the informational environment and not the underlying project cash flows, optimal contracts independent of investor sophistication and entrepreneur's private benefit achieves the first best: the entrepreneur issues warrants in the initial round for purchasing convertible securities later, then raises the remaining investment by selling residual claims to competitive outsiders. We further characterize conditions for equity, debt, and call option to be optimal, and demonstrate robustness of our findings, for example under scalable investment and partial commitment to information design.

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1 Introduction

What is the benefit of raising capital from intermediaries an entrepreneur repeatedly interacts with instead of issuing securities in public markets? As it is well-known in the literature on relationship banking, through monitoring and interpreting interim information, intermediaries can mitigate informational asymmetry and moral hazard when they form relationship with entrepreneurs to become "insider" financiers (e.g. Diamond (1984, 1991), Fama (1985), Ramakrishnan and Thakor (1984)).[1] Yet, an early insider may hold up entrepreneurs (Sharpe (1990) and Rajan (1992)). Similarly, it is well-recognized that venture capital enables entrepreneurial experimentation that generates information about the start-ups, and investors stage investments to learn and manage continuation decisions (Kerr, Nanda, and Rhodes-Kropf (2014)). But at the same time, staged financing can produce conflicts of interest and hold-ups (Admati and Pfleiderer (1994) and Ewens, Rhodes-Kropf, and Strebulaev (2016)) and give disproportionate bargaining power to the initial venture capitalist (Fluck, Garrison, and Myers (2006)).

Existing studies typically focus on the borrower or entrepreneur's actions that shape project cash flows and abstract away from those that alter the informational environment. Consequently, information available to insider investors is mostly exogenous. Yet in reality entrepreneurs' experimentations are endogenous, and the interim information produced not only depends on the funding available, but also on the entrepreneur's desire to persuade investors to continue financing. One example is Skycatch, Inc., an end-to-end solution for data capture using drones.[2] To raise capital from incumbent investors and send positive signals to other potential investors in 2015, it deliberately chose to service a large, well-established client for a more challenging job over performing smaller trial projects. Both tasks would consume about the same resources at the time, but the former is more difficult but produces more extreme signals that, when successful, helps the firm to "stand out from the rest of drone-related startups".[3] Another example is the beta launch for software and games as a form of experimentation. Soft launching in Canada, as Minecraft did, presents a less risky experiment, whereas launching in the US costs about the same but could generate more extreme signals because the large population and excessive media attention could spoil

[1] We use "financier" and "investor" interchangeably.

[2] With a post-money valuation of $96 million, it is among the most valuable enterprise startups in 2016 according to pitchbook.com, a company that offers a database of venture and private equity funding.

[3] According to one of their investors at Amino Capital. See also "The 38 most valuable enterprise startups of 2016", Business Insider, Dec 22, 2016.

the actual global launch later.[4] Career concerns and managerial choices of projects exhibit similar features.

Several questions naturally arise. Do the source and endogenous production of information matter for sequential fund-raising? Do they affect the link between bank orientation and competition? Do financially constrained entrepreneurs and established firms conduct experiments differently? What are their implications for designing securities for sequential investors? Motivated by these questions, this paper studies a Beyasian persuasion game with an initial investment stage and ex-post contingent transfers, and by doing so, represents a first attempt to endogenize information design in relationship financing and analyze the associated hold-up problem.

We show that the entrepreneur's endogenous information production reduces an insider's rent from her interim bargaining power, but is inefficient and holds up her initial investment. Relationship investors' independent information and interim competition mitigate the problem, and can either complement or substitute the entrepreneur's experimentation. We then offer a contractual solution to fully resolve the entrepreneur's moral hazard of information production. Unlike the case of information monopoly and hold-up problem in relationship lending, dynamic debt contracts with termination clauses in Boot (2000) and Von Thadden (1995) are insufficient. Instead, the entrepreneur optimally promises early insider investors the option to purchase convertible securities in future at pre-specified price and quantity, and upon the insider's continued financing, issues residual securities to outsider investors.

Our theory is immediately relevant to two major areas of finance. First, it underscores the impact of endogenous information production and clarifies its interactions with investor sophistication and competition, rationalizing empirical patterns in *relationship lending* that extant theories could not account for. Second, it demonstrates the robustness of convertible securities in the *venture capital*, and derives the optimal contract that is consistent with real-life practice. More importantly, given that the solutions to many of the world's biggest problems such as Alzheimer's disease, global warming, and fossil-fuel depletion require large initial funding, reliable financing relationship, and long-term experimentation (Nanda and Rhodes-Kropf (2013) and Hull, Lo, and Stein (2017)), the cost of inefficient information production could be tremendous. Our study helps underscore and formalize this practical issue, and develops potential contractual solutions. From a theory perspective, our study sheds light on Bayesian Persuasion games with hold-ups and contingent transfers, and deepens

[4] "Why Is Canada Such A Good Testing Ground For Game Releases?" Forbes Tech. Nov 27, 2012

our understanding of contracting under the moral hazard of information production, which significantly differs from other moral hazards.

Specifically, our baseline model considers a capital-constrained entrepreneur with a project that requires two rounds of financing. The first round requires a fixed investment that enables the entrepreneur to "experiment"—broadly interpreted as conducting early-stage activities such as hiring key personnels, acquiring initial users, and developing product prototypes. An investor in the first round, whom we call an insider, does not have expertise on project experimentation and the entrepreneur lacks ex-ante commitment to any specific production technology of interim information, and hence they cannot fully contract on experimentation. But by monitoring and having insider access, the relationship investor observes interim signals from the experiment that is informative of the eventual profitability.

After forming the financing relationship with an investor in the first round, the entrepreneur raises additional capital in a second round by issuing securities to this insider and potentially outsider investors. Because the insider investor has information monopoly, it is well-known that she can more efficiently continue or terminate the project, relative to arms-length investors (outsiders), if there exists any; she can also hold up the entrepreneur to extract more rents, which distorts the latter's incentives (e.g., Rajan (1992) and Gorton and Winton (2003)). In the context of venture financing, early insider investors often obtain favorable terms in the second round such as the right to first refusal, or conversions at discount, which also distinguishes her from outsider investors. Following standard persuasion games that feature divergent objectives of the sender and the receiver, We assume that the entrepreneur enjoys private benefit of continuing the project that is difficult to verify or contract upon (Dyck and Zingales (2004); Aghion and Bolton (1992)).

We first show that when the entrepreneur endogenously experiments, he follows a threshold strategy and generates a continuation signal if the true cash flow from the project is sufficiently high. Akin to the soft-budget constraint problem, through designing the informational environment and providing vague signals, the entrepreneur makes the insider investor indifferent between termination and continuation and just break even. On the one hand, this reduces distortions due to the insider's bargaining power, for example, from her information monopoly. On the other hand, the entrepreneur's interim information production is inefficient, rendering the insider incapable of recovering the initial investment in forming the relationship. Projects may not get financed in the first round to start with. These results are in sharp contrast to existing theories on bank monitoring that ignore the endogenous experimentation and thus the endogenous informational environment, and point to the importance of the moral hazard of information production.

For "sophisticated" investors, who are in industries requiring less entrepreneur-specific knowledge or are very skilled at evaluating a project using independent information, they can extract positive interim rent as insiders, partially restoring the feasibility of relationship financing. Furthermore, their sophistication and the entrepreneur's information production are typically complements, but as investor sophistication increases, there can be a "crowding-out effect on entrepreneur's information production. The intuition is that if the investor has highly informative independent signals about the profitability of the project, the entrepreneur prefers to provide less clear information to increase the chance of inefficient continuation for his private benefit. Therefore, the social efficiency is non-monotone in the investor sophistication.

Relationship lending also becomes viable with moderate interim competition, because selling to competitive outsiders encourage more efficient information production by the entrepreneur. Investor competition (reflected through the insider's interim bargaining power) and sophistication (captured by the informativeness of her independent signal) jointly impact the dependence of relationship financing on competition, yielding non-monotone patterns overall. In particular, for intermediate levels of investor sophistication, the ease of relationship formation proxied by the initial funding capacity can therefore depend on interim competition in a U-shaped pattern, consistent with empirical findings that extant theories cannot explain (Elsas (2005) and Degryse and Ongena (2007)).

The moral hazard of information production manifests itself in other forms of security as well, and the economic insights apply beyond relationship lending and staged venture financing. It is therefore important to explore contractual solutions to this general problem. Moreover, in reality, a designer may not always know the entrepreneur's private benefit and the investor sophistication ex ante. As such, we first show that contractual solutions in earlier studies do not solve the problem. We then take a step further to derive the robust optimal design of securities which entails the entrepreneur optimally giving early insiders warrants to purchase some form of convertible securities at pre-specified price and quantity, and issuing residual securities such as equities to outsiders, consistent with the practice in venture financing.

Intuitively, the entrepreneur is biased towards continuation and overall gets all the ex-ante surplus. At the time of forming a financing relationship, he wants to but typically cannot commit to efficient information production in the interim. An optimal security therefore should facilitate such a commitment, balancing the entrepreneur's payoff sensitivity to his information production either indirectly through the insider's continuation decision (continuation channel) or directly through internalizing the cost of inefficient continuation himself (payoff channel). The problem involves infinitely-dimensional nested optimization, and the solution methodologies in conventional moral hazard problems do not apply. We instead take a constructive-proof approach. Our key insight is that unlike the conventional moral-hazard models, the entrepreneur's action does not affect the distribution of cash flows, but instead affects the informational environment which is independent. Contingencies on both continuation decisions and payoffs upon continuation can then perfectly penalize inefficient information production, achieving the first-best outcome that is often infeasible under traditional moral hazards except for some special cases.

Specifically, we explicitly construct an optimal contract that restores social efficiency. To do this, first note that because the insider investor cannot fully internalize the entrepreneur's private benefit of continuation, the continuation channel is sensitive to investor sophistication. An entrepreneur thus finds it more effective and robust to reserve a large enough proportion of second-round financing for competitive outsiders, so that the payoff channel dominates. By giving the insider debt-like securities in bad states of the world, convertible securities maximize the entrepreneur's exposure to the cost of inefficient continuation. Internalizing this cost leads to more efficient information production. At the same time, relationship financing is feasible as long as the contract yields the insider enough interim rent in order to recover the investment in the initial round, leaving the security design indeterminate in good states of the world. This design using warrants that specify both the quantity and terms for the insider to purchase convertible securities is the only one robust to investor sophistication and entrepreneurial bias for continuation.

Finally, we discuss the optimality of other securities, partial commitment to information design, security design by the insider, scalable investment, among others. In addition, we further explore investor sophistication and the interval nature of information design, which has implications on conventional assumptions in models of information disclosure and speculation in financial markets.

Literature

Our paper foremost relates to the enormous literature on relationship financing. Boot (2000), Gorton and Winton (2003), and Srinivasan et al. (2014) survey relationship lending. Theoretical studies on relationship banking focus on information production and control (e.g., Diamond (1984, 1991) and Fama (1985)): while relationship financing can be more efficient (e.g., Petersen and Rajan (1994)) than borrowing from arm's length lenders, it naturally gives the financiers information monopoly (Berger and Udell (1995), Petersen and Rajan (2002), and Rajan (1992)). This information monopoly leads to potential hold-ups in relationship lending (e.g., Santos and Winton (2008) and Schenone (2010)) and venture capital (Burkart, Gromb, and Panunzi (1997) and Ewens, Rhodes-Kropf, and Strebulaev (2016)). We add to the debate by endogenizing the informational environment and highlighting the moral hazard of information production.

Such endogenous information production in persuasion games can lead to a failure of the initial financing, and is thus related to the classical hold-up problem in Hart and Moore (1988). The contractual solution offered in Hart and Moore (1988), Aghion, Dewatripont, and Rey (1994), and Nöldeke and Schmidt (1995) do not induce first-best outcomes in our setting because of limited liability and contingent transfer. Two other related papers are Von Thadden (1995) and Nöldeke and Schmidt (1998). Von Thadden (1995) considers dynamic financing relationship and similarly uses option contracts to solve the problem of information monopoly; Nöldeke and Schmidt (1998) study two parties making sequential relationship-specific investments and show that for sufficiently low uncertainty of project outcome, an option for purchasing ownership implements the first-best. We endogenize interim information for general project uncertainties and show that long-term contracts with specific security choices are needed to restore efficient information production. Our solution also differs in its robustness: it does not require differential contracts for various levels of entrepreneurial bias and investor sophistication.

Our paper is also broadly related to the role of intermediaries and security design and contracting in financing innovation (Da Rin, Hellmann, and Puri (2011) and Kortum and Lerner (2001)). We theoretically show how informed financier promotes innovation (see Herrera and Minetti (2007) for empirical evidence), allowing endogenous experimentation, investor sophistication, and competition. In particular, the insider investor's sophistication can have non-monotone effects on overall informational efficiency. Several studies including Bergemann and Hege (1998) and Hörner and Samuelson (2013) focus on costly and uncontractible effort that affects learning in an exogenously given manner, and the dynamic agency cost it induces, whereas we examine information design which is observable but uncontractible ex ante. Furthermore, we add to earlier studies on the extensive use and optimality of convertible securities (e.g., Gompers (1997), Kaplan and Strömberg (2004), and Hellmann (2006)) by endogenizing information design.[5] We differ in our focus on en-

[5]Complementary is Yang and Zeng (2017) which shows that under flexible information acquisition by

trepreneur's endogenousinformation production and the financing relationship. To our best knowledge, we are also the first to show that issuing warrants to buy convertible securities for early insider investors and residual claims for later outsider investors is optimal and robust.

Furthermore, the effect of competition on bank orientation has received significant attention but reamins ambiguous from a theoretical perspective. Petersen and Rajan (1995) and Dell'Ariccia and Marquez (2004) argue that it is harder to form initial lending relationship when interim competition is bigger; Boot and Thakor (2000) and Dinc (2000) derive the opposite when considering both relational and transactional lending; Yafeh and Yosha (2001) and Anand and Galetovic (2006)) suggest that competition can have ambiguous effects on lending relationships, but typically predict an inverted U-shape pattern. Yet empirically, Elsas (2005) and Degryse and Ongena (2007) document a U-shaped effect of market concentration on relationship lending which cannot be explained by extant theory. Our model offers an explanation of the empirical pattern and highlights the integral role of the investor's relative sophistication.

From a theory perspective, our paper sheds new lights on contracting under moral hazard frictions. Studies following the seminal studies of Holmstrom (1979) and Innes (1990) mostly concern agents' actions that alter the distribution of cash flows only; though extant studies on the agency issues in costly experimentation such as Hörner and Samuelson (2013) and Bergemann and Hege (1998) consider information-acquisition effort, the principal cannot isolate information produced by the agent (hidden effort and hidden information). Our findings therefore highlight the difference between the moral hazard of information production and traditional moral hazards of effort provision. Because the entrepreneur in our setting shapes the informational environment for decision making instead of the distribution of project cash flows, and in a way orthogonal to other channels of information production (uncontractible effort and observation information), the continuation outcome perfectly reveals whether the entrepreneur's action is efficient, and can be contracted upon to incentivize *first-best* effort — an outcome unattainable in conventional settings. Our solution approach using constructive-proof and robustness argument also differs from earlier studies and can be potentially applied to similar problems.

Finally, our paper contributes to the field of information design (Bergemann and Morris (2017) and Hörner and Skrzypacz (2016)), especially Bayesian Persuasion (Kamenica and Gentzkow (2011) and Ely (2017)). We take a linear programming approach similar

investors and when information is valuable, a combination of debt and equity is optimal for the entrepreneur.

to Bergemann and Morris (2017), but allow infinite payoff-relevant states and privately informed receivers. We also relax the assumption that the sender's utility from a message completely depends on the expected state (Kolotilin (2017)), or the sender's payoff over the receiver's actions is independent of the state (Gentzkow and Kamenica (2016)). Kolotilin (2017) similarly finds a non-monotone effect of the receiver's signal precision (our investor sophistication) on sender and receiver's utilities; featuring both sender's and receiver's payoffs dependent on the state and receiver's type, Guo and Shmaya (2017) characterize the interval structure of sender-optimal design under more general settings. None of these studies concern contracting under the moral hazard of information production. Most closely related to our paper is Szydlowski (2016), which derives an irrelevance result of security choice when the entrepreneur jointly designs disclosure and security.[6] Both Szydlowski (2016) and our paper feature security choices endogenizing the dependence of the sender and receiver' payoffs on the state, which is new to the literature. Our paper is the first to apply information design (and jointly with security design) to relationship financing, and discuss optimal securities for sequential financing under endogenous informational environment.

2 Relationship Financing and Information

This section describes the basic economic and informational environment. We first rederive the benchmark results from the existing literature that takes information as exogenous: mitigation of informational asymmetry, which leads to more efficient financing, and the hold-up problem due to the insider's bargaining power, for example, from her information monopoly. We then show that with endogenous information production, the relationship financier faces a "reverse hold-up" problem due to the entrepreneur's **moral hazard of information production (MIP)**.

2.1 Model Set-up

A risk-neutral entrepreneur has a project that requires a fixed investment $I \in (0, 1)$, and produces an uncertain cash-flow $X \in [0, 1]$ with a prior distribution denoted by a continuous and atomless pdf $f(X)$. In the baseline, the entrepreneur issues short-term debt to finance the project in multiple stages. In the baseline model, the terms of financing is negotiated,

[6] Other emerging studies applying information design in finance involve capital structure (Trigilia (2017)), dynamic government intervention (Cong, Grenadier, and Hu (2017)), and stress tests (Bouvard, Chaigneau, and Motta (2015), Goldstein and Leitner (2015), and Orlov, Zryumov, and Skrzypacz (2017)).

which means the contracts are short-term, in line with the literature on relationship lending. The entrepreneur receives private benefit $\varepsilon \in (0, \bar{\varepsilon})$, for some $\bar{\varepsilon} < I$, if the project is financed, which could correspond to his utility from control ("nonassignable control rent" in Diamond (1993), see also Winton and Yerramilli (2008); Szydlowski (2016)), or payoff from assets- or business-in-place (Myers and Majluf (1984)), or perquisites managers appropriate (Jensen and Meckling (1976)). The private benefit is significant, but is in reality hard to quantify or verify (Dyck and Zingales (2004); Aghion and Bolton (1992)). We therefore assume that it is non-contractible.[7] There is no time discounting.

To best illustrate our economic mechanism and match reality for early business startups and financing of innovation, we assume $\mathbb{E}[X - I] < 0$, i.e., the project would not be financed directly by arms-length financiers ex ante, a case for which relationship financing and informational considerations are the most important. However, with investment $K > 0$ the project can go through an initial stage of monitored development – experimentation that generates more information about the distribution of final cash-flow. The investor who invests K forms a financing relationship with the entrepreneur, monitors, and observes this interim information to better decide whether to continue financing.

One may think of $K + I$ as the total investment needed, but raised in stages whereby early experimentation generates interim information (Kerr, Nanda, and Rhodes-Kropf (2014)). For simplicity, we assume the seed investment K generates negligible initial cash flows compared to the final payoff expected by the investor and the entrepreneur, which is similar to normalizing the liquidation value to zero (Diamond (1993)) and can be equivalently interpreted as K representing the investment net of the initial cash flows. Furthermore, $\mathbb{E}[(X - I)\mathbb{I}\{X \geq I - \bar{\varepsilon}\}] \geq K$, i.e., it is worthwhile for a social planner to experiment to decide on continued financing.

Informational Environment

From the entrepreneur's experimentation in the earlier stages, the investor receives some useful interim information for the continuation decision. In our model, an "experiment" generates a signal $z \in \mathcal{Z}$, whose distribution follows from a mapping $\pi : [0, 1] \to \Delta(\mathcal{Z})$. Therefore, $(\mathcal{Z}, \pi)$ fully characterizes the experiment, and $\pi(z|X)$ shows the conditional probability

[7] For example, the investor cannot promise to pay ε upon terminating the project, because otherwise it would lead to entry of fly-by-night firms. Such an arrangement not only leads to adverse selection when ε is privately know by the entrepreneur, but also leads to equilibrium multiplicity and indeterminacy of the informational environment.

that z is realized when the state is X.

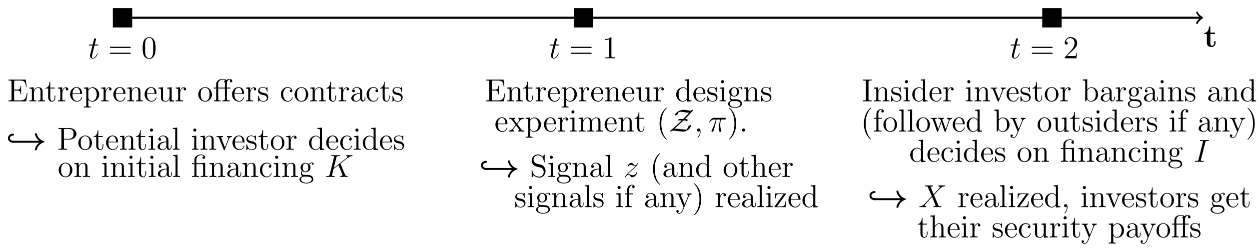
Prior studies almost uniformly treat the interim information generation, $(\mathcal{Z}, \pi)$, as exogenous. While outsider investors may potentially know $(\mathcal{Z}, \pi)$, the entrepreneur cannot credibly communicate the signal z to outsiders. We depart from the existing literature by endogenizing the experimentation and thus information production. It may appear unrealistic at first to assume that $(\mathcal{Z}, \pi)$ can be arbitrarily informative, especially that the entrepreneur's experiment may have real consequences on hiring or prototype development. However, the assumption is innocuous under the interpretation of X as the most informative signal the entrepreneur can generate while conducting its usual entrepreneurial activities (Kamenica and Gentzkow (2011)). Furthermore, even though experimentation costs differ significantly across sectors and industries (Nanda and Rhodes-Kropf (2015)), the dispersion within a product category is much smaller, and the cost of experimentation has declined dramatically, particularly in industries such as software and digital media that have benefited from the advent of the Internet, because fixed investments in infrastructure and hardware are no longer necessary.[8] Therefore, experimentation in our setup can be interpreted as either concerning these industries, or entrepreneurial activities with similar costs but great flexibility in terms of the information structure it delivers.

In the baseline model, we assume that only the entrepreneur has the relevant skill and expertise to design $(\mathcal{Z}, \pi)$ after raising K, which happens when the lender either has no previous experience on the project or it is too costly for him to extract information, e.g. the firm is located in a hardly accessible location, or the investor has no relevant expertise to generate independent signals. Importantly, the design space is rich and complex such that the entrepreneur cannot commit to the design ex ante due to contract incompleteness.[9] That said, intermediaries such as banks have the ability to monitor the experiment and interpret the results once they finance the entrepreneur initially, giving them insider information monopoly. The model thus applies to situations where not only is the experiment choice observable to the insider financier, and realizations are also directly observable and verifiable ex post, which are common in the Bayesian persuasion literature (e.g., Kamenica and Gentzkow (2011)).

[8] See, e.g., Kerr, Nanda, and Rhodes-Kropf (2014), Palmer (2012), and Blacharski (2013). Industry observers suggest that firms in these sectors that would have cost \$5 million to set up a decade ago can be done for under \$10,000 today through renting in tiny increments from cloud computing providers and efficiently scaling up as demand for their products increases.

[9] For example, an angel investor does not know what kinds of team members a founder is going to assemble, or what kind of field-specific experiments to conduct. Carroll (2015) considers a similar situation but with the alternative assumption that investors care about robustness.

Figure 1: Timeline of the game.



The game has three periods, $t = 0, 1, 2$. In period 0, the investor decides whether to become an insider and finance the experiment. After getting K in period 0, the entrepreneur in period 1 chooses an observable experiment $(\mathcal{Z}, \pi)$ which then generates signal $z \in \mathcal{Z}$ about the profitability of the project. Then, the investor bargains with the entrepreneur and decides whether to follow on with invest I. Without interim competition, the insider has the whole bargaining power, as is typical in information monopoly settings (e.g., Rajan (1992)). In other words, she requires a high interest rate to get the whole interim surplus from the investment. Finally in period 2, the project cash-flow X is realized and observed. Figure 1 summarizes the time line. In this Section, we focus on the case without interim competition.

2.2 Equilibrium

Because the insider investor can negotiate the terms of the short-term debt contract and has all the bargaining power, she effectively sets the face value or interest sufficiently high such that she gets all the interim payoff X.

Given an experiment $(\mathcal{Z}, \pi)$, the insider investor decides to invest after observing $z \in \mathcal{Z}$ if $\mathbb{E}[X|z] \geq I$ Let $\mathcal{Z}^+$ denote the set of all signals in $\mathcal{Z}$ that induce investment, i.e.

$$\mathcal{Z}^+ = \{z \in \mathcal{Z} | \mathbb{E}[X|z] \geq I\}$$

For each possible value of outcome X, we can find the probability of investment implied by $(\mathcal{Z}, \pi)$, which we refer to as the "*investment function*":

$$\mathcal{I}_\pi(X) = \int_{z \in \mathcal{Z}} \mathbb{I}_{\{z \in \mathcal{Z}^+\}} \pi(z|X) dz.$$

Benchmark with Exogenous Information

Consider the benchmark case of exogenous interim information, which earlier studies specialize to. When there is no interim competition, the entrepreneur in periods 1 and 2 is completely locked-in with the insider investor who finances K. The insider investor's information monopoly gives her full bargaining power over the interim surplus generated by experiment, $\mathbb{E}[(X-I)\mathcal{I}_\pi(X)]$, distorting the entrepreneur's uncontractible effort as in earlier models such as Rajan (1992) — the traditional information hold-up problem.[10]

The expected payoffs for the investor and the entrepreneur are

$$U^I(\mathcal{Z},\pi) = -K + \mathbb{E}[(X-I)\mathcal{I}_\pi(X)] \tag{1}$$

$$U^E(\mathcal{Z},\pi) = \varepsilon\mathbb{E}[\mathcal{I}_\pi(X)] \tag{2}$$

Moreover, financing relationship is feasible if and only if the experiment is sufficiently profitable for the insider to justify the initial investment in the relationship, i.e, $\mathbb{E}[(X - I)\mathcal{I}_\pi(X)] \geq K$. If the experiment is not informative enough, the interim rent would be too low for the relationship financing to be feasible initially (e.g., Nanda and Rhodes-Kropf (2013)). One extreme example involves $\pi(z|X)$ being independent of X, where relationship financing is futile. Another extreme example is when the experiment is fully informative, i.e. $\mathcal{Z} = \{h, l\}$ and $\pi(h|X) = 1$ iff $X - I \geq 0$, relationship financing is always feasible.

Endogenous Experimentation and Information

Next, we consider the case that the entrepreneur designs the experiment endogenously to maximize his expected payoff. Lemma 1 characterizes the optimal experimentation and the investment function it implements:

Lemma 1 (Endogenous Information)**.** *There is an optimal experiment with two signals, i.e.,* $|\mathcal{Z}| = 2$*, and threshold* $\bar{X}$ *that solves*

$$\mathbb{E}[X|X \geq \bar{X}] - I = 0 \tag{3}$$

The experiment generates a high signal h *and induces investment if* $X > \bar{X}$*; otherwise, it generates a low signal* l*, leading to termination. Moreover, all optimal experiments implement the same investment function and payoffs, rendering the equilibrium essentially unique.*

[10] Here we focus on information production and abstract away from entrepreneur's effort provision that improves project cash flows. Section 5.6 discusses their interactions in more details.

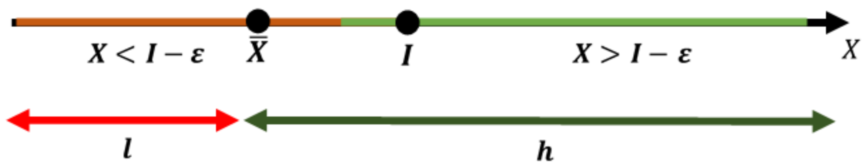


Figure 2: The optimal information design. Lighter green line ($X > I - \epsilon$ indicates the range of outputs that investment is socially optimal. Darker green line ($X > \bar{X}$) indicates the the range of outputs that the investment takes place in the subgame equilibrium at $t = 1$ because the experiment generates signal h.

Lemma 1 characterizes the equilibrium investment function conditional on the investor's funding the experiment at $t = 0$. It reiterates Bergemann and Morris (2017)'s point that we can restrict attention to information structures where the players' type sets are equal to their action sets, reminiscent of the direct revelation principle. In equilibrium, while all profitable projects are financed, some inefficient ones are financed as well due to MIP, as it is clear from (3) that $\bar{X} < I$. Figure 2 illustrates this optimal experiment design. Corollary 1 concludes that for small enough values of K, the implemented investment function improves upon the case without experimentation and learning.

Corollary 1. *Under endogenous information production, it is socially efficient to start a relationship financing iff* $\varepsilon(1 - F(\bar{X})) > K$.

This result is a reminder that relationship financing can improve interim efficiency, because the lender acquires specific information about the borrower in earlier rounds of financing.

That said, Equation (3) also implies that the investor cannot get positive payoff in the subsequent game if she pays K at $t = 0$, despite having the full bargaining power. Intuitively, the entrepreneur designs the high signal such that the investor is indifferent between continuation and termination, while the entrepreneur always strictly prefers continuing the project. Anticipating this subgame payoff, the investor has no incentive to pay $K > 0$ to start with. The next proposition states this "reverse hold-up" problem due to MIP.

Proposition 1 (Financing Breakdown due to MIP)**.** *In the unique equilibrium, no project can be financed in the initial round unless* $K = 0$.

Surprisingly, even with full bargaining power, for example, through information monopoly, the insider financier is held-up, resembling a problem of soft-budget constraint (e.g., Dewatripont and Maskin (1995)). Proposition 1 emphasizes the inefficiency that arises from the agency problem in information production and it cannot be resolved by debt financing from insider investors. This result shows that even though the investor acquires information through monitoring and the entrepreneur is completely locked-in with the investor, if the entrepreneur endogenously produces information in earlier stages, no rent would accrue to the investor. It should be apparent now that taking information as exogenous in relational financing is not an innocuous assumption.

To highlight MIP's stark effect, we have abstracted away from private information the insider has and interim investor competition. Section 3 takes these factors into consideration, and show that the insider investor gets positive payoff in the second round if she acquires independent information or faces outsider competition. Nevertheless, MIP persists, and is robust to commitment to no-renegotiation, scalable investment, and to alternative allocation of security design right, as discussed later.

3 Investor Sophistication and Interim Competition

So far the insider investor in our model relies solely on the entrepreneur's experimentation for learning. In reality, an insider investor may have information independent from the entrepreneur's information production. For example, the investor may experiment to some extent, or use her proprietary business experience and expertise to better predict the market demand of the product, or project valuation in future financing rounds. Moreover, physical proximity between the lender and the borrower facilitates information collection and relational lending (e.g., Agarwal and Hauswald (2010); Mian (2006)), so a closely located insider may have proprietary knowledge about the local economy. We collectively refer to the ability of a relationship financier to utilize such information as "*investor sophistication*".

In this section, we first analyze how investor sophistication mitigates MIP, and then allow interim investor competition, before showing how the interaction of MIP with both investor sophistication and competition can help us rationalize puzzling empirical patterns in the formation of lending relationship.

3.1 Investor Sophistication

We first show that investor sophistication generates a positive rent for the insider, which increases the feasibility of forming a lending relationship. Moreover, when the investor is not so sophisticated in producing high-quality signal, investor sophistication and entrepreneur's information production are complements, and in equilibrium she discards her independent information and invests based only on the signal generated by the entrepreneur's experiment.

To highlight economic intuition and ease exposition, we model investor sophistication by simply assuming that after the entrepreneur's experiment, the insider investor also receives signal $y \in \{\tilde{h}, \tilde{l}\}$ with specific signal structure $(\{\tilde{h}, \tilde{l}\}, \omega_q)$ for some $q \in (\frac{1}{2}, 1)$, where

$$\omega_q(\tilde{h}|X) = \begin{cases} q & I \leq X \leq 1 \\ 1-q & 0 \leq X < I. \end{cases}$$

In other words, the investor receives a high signal $y = \tilde{h}$ with probability $q > \frac{1}{2}$ if the project is profitable, and with probability $1 - q < \frac{1}{2}$ otherwise.

As we discuss in the proof in the appendix, many results generalize to the richer specification where the set of insider's independent information structure is finite, and is ranked by the likelihood of inducing continuation for the same signal z. In the current special case, q is common knowledge and captures the investor's ability in independent experimentation, namely the degree of the investor's sophistication *relative* to the entrepreneur's and the fundamental uncertainty. q can be related to the physical distance of the investor from the borrower or the extent to which the project is field-specific or technical. A higher q is naturally associated with relatively less advanced technologies the entrepreneur has expertise on.

We also assume, unless stated otherwise,

Assumption 1. *The insider's private signal is only moderately informative, i.e., $q < \hat{q}$, where $\hat{q}$ is the positive root of $q^2 + q = 1$.*

This assumption allows us to focus on simple forms of optimal experimentation in our baseline model. In section 5.3, we show our main results all go through for $q > \hat{q}$, with the caveat that the optimal experimentation takes a more complicated form. Social welfare is non-monotone in the investor sophistication. When the investor becomes too sophisticated, the entrepreneur may prefer sending a vaguer signal to increase his chance of project continuation which may be inefficient. Hence the endogenous information and investor sophistication can be substitutes too.

Equilibrium Characterization

Altogether, the investor receives two signals before the continuation decision. One is z generated by the entrepreneur's experiment $(\mathcal{Z}, \pi)$ and the other one is y. She invests if

$\mathbb{E}[X|z,y] \geq I$. One can see that the investor invests after receiving $(z,\tilde{h})$, for some $z \in \mathcal{Z}$, if

$$(1-q)\int_0^I (X-I)\pi(z|X)f(X)dX + q\int_I^1 (X-I)\pi(z|X)f(X)dX \geq 0;$$

and after receiving $(z,\tilde{l})$, she invests if

$$q\int_0^I (X-I)\pi(z|X)f(X)dX + (1-q)\int_I^1 (X-I)\pi(z|X)f(X)dX \geq 0.$$

Under this information structure the investor gets positive interim rent, making her more willing to bear the initial cost K. The intuition is the following: the realization of y and its information structure are outside the entrepreneur's control. If $(\mathcal{Z},\pi)$ is designed in a way that for some $z \in \mathcal{Z}$ the investor invests if she receives $(z,\tilde{l})$, then she would invest and get positive expected profit if $(z,\tilde{h})$ is realized because $\mathbb{E}[X|z,\tilde{h}] > \mathbb{E}[X|z,\tilde{l}] \geq I$. Therefore, if the investor invests with positive probability after receiving $\tilde{l}$, she gets positive expected profit in the second round. We now characterize the endogenous experiment and information production in the subgame after the investor pays K.

Lemma 2 (Investor Sophistication)**.** *Denote the equilibrium investment function by $\mathcal{I}^q(X)$.*

(a) There is an optimal experiment with $|\mathcal{Z}| = 2$ and threshold $\bar{X}(q) \in (\bar{X}, I)$ that solves

$$q\int_{\bar{X}(q)}^I (X-I)f(X)dX + (1-q)\int_I^1 (X-I)f(X)dX = 0 \tag{4}$$

It sends a high signal if $X > \bar{X}(q)$ and a low signal otherwise. Moreover, all optimal experiments implement the same information structure and equilibrium payoffs.

(b) Denote $\bar{K}_Q(q) \equiv \mathbb{E}[(X-I)\mathcal{I}^q(X)]$ the highest initial cost an insider can bear to rationally start a financing relationship. Then, $\bar{K}_Q(q)$ is locally increasing in $[\frac{1}{2},\hat{q}]$.[11]

To understand part (a), note that the existence of signal y influences the way the entrepreneur designs the experiment. In particular, the entrepreneur sends a more accurate information to preempt the risk of loosing high payoffs when the insider receives $\tilde{l}$. When q is low, the risk of such loss is higher.[12] Figure 3 illustrates the optimal endogenous ex-

[11]We show later in section 5.3 that the insider's rent is not globally monotone over the interval $[\frac{1}{2},q]$.

[12]To see this effect more clearly, consider another information structure for the investor's information, in which she receives a perfectly informative signal with probability $r > 0$ and an uninformative signal with probability $1-r$. In this case, the entrepreneur knows if a perfectly informative signal is realized, the investor does not use the signal provided by the entrepreneur's experiment. In other words, he is not afraid of losing a high profit as a result of an error in the investor's signal. Therefore, the entrepreneur only targets the case

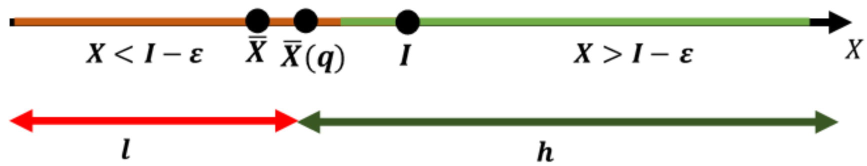


Figure 3: The optimal information design with sophisticated investors, for $q \in (\frac{1}{2}, \hat{q}]$. Lighter green line ($X > I - \epsilon$) indicates the range of outputs that investment is socially optimal. Darker green line ($X > \bar{X}(q)$) indicates the the range of outputs that the investment takes place in the subgame equilibrium at $t = 1$.

perimentation under sophisticated insiders. Investor sophistication and the entrepreneur's endogenous information production are complements.

Lemma 2(b) conveys the message that the investor sophistication generates positive profit and up to some level, the more sophisticated the insider investor is, the more socially efficient investment decisions are. Investor sophistication in this way mitigates the inefficiency caused by MIP.

3.2 Interim Investor Competition

Next, we focus on the role of interim inter-bank or credit market competition, by temporarily shutting down investor sophistication. As mentioned in Section 2, this situation corresponds to financing technically advanced or innovative projects, or financing by investors so inexperienced in evaluating the project that entrepreneurs fully control information production for the continuation decision.

Earlier studies typically model bank competition in reduced-form. For example, in Petersen and Rajan (1995), a bank's market power directly relates to the interest rate it charges; in Rajan (1992), a bank's "bargaining power" pins down the share of the unallocated surplus it gets upon continuation; in Hauswald and Marquez (2003), other outsider financiers may observe part of any interim information generated with small cost or freely; in Diamond (1991), the entrepreneur establishes some observable track records. We similarly model an insider investor's market power as the share μ it gets from the surplus upon continuation, $\mathbb{E}[X - I|z]$. The expected payoffs for the entrepreneur and the insider investor after the

that the investor receives the uninformative signal. Consequently, the experiment is as described in Lemma 1.

realization of signal $z \in \mathcal{Z}$ are then:

$$\begin{aligned} U^E(z;(\mathcal{Z},\pi)) &= \varepsilon \int_0^1 \mathcal{I}(X) f(X) dX + (1-\mu) \int_0^1 (X-I)\mathcal{I}(X) f(X) dX \\ U^I(z;(\mathcal{Z},\pi)) &= -K + \mu \int_0^1 (X-I)\mathcal{I}(X) f(X) dX \end{aligned} \tag{5}$$

Note that the insider investor needs to earn positive profit in expectation to recover the initial cost K. We know from Lemma 1 that she recovers nothing in expectation when $\mu = 1$, i.e., when she has the highest degree of bargaining power. Interestingly, she can get more interim rent when her bargaining power is moderate, as shown in Lemma 3, because an intermediate value of μ helps the entrepreneur internalize the cost of inefficient continuation. Hence, he provides better information to the insider and decreases the probability of inefficient continuation. The increase in total surplus in the interim can outweigh the insider's loss from claiming a smaller share of the surplus, facilitating the formation of financing relationship.

Lemma 3 (Interim Competition). *Denote $\mathcal{I}^\mu(X)$ as the equilibrium investment function given the level of interim competition μ.*

(a) Under interim competition, the entrepreneur's optimal experiment entails two signals and produces high signal h when $X > \max\{\bar{X}, I - \frac{\varepsilon}{1-\mu}\}$, where $\bar{X}$ is specified in (3). Therefore $\mathcal{I}^\mu(X) = \mathbb{I}_{\{X \geq \max\{\bar{X}, I - \frac{\varepsilon}{1-\mu}\}\}}$.

(b) The expected interim rent the insider gets, $\bar{K}_M^\varepsilon(\mu) \equiv \mu \mathbb{E}[(X-I)\mathcal{I}^\mu(X)]$, is non-monotone in μ. Specifically, it is unimodal for small enough values of ε.

Part (a) implies that interim competition only matters when it is strong enough, i.e., $\mu < 1 - \frac{\varepsilon}{I-\bar{X}}$. Otherwise, the equilibrium experiment, investment function, and payoffs remain the same as those in Lemma 1. Note that similar to $\bar{K}_Q(q)$, $\bar{K}_M^\varepsilon(\mu)$ indicates the capacity for forming a financing relationship for a given (ε, μ). Absent investor sophistication, increasing interim competition increases this capacity first and then reduces it.

Sometimes the level of competition μ is not entirely exogenous (Dell'Ariccia and Marquez (2004)), for example, in the case of investor syndication or commitment to credit market competition. We can interpret the information spillover described in Hauswald and Marquez (2003) as endogenized by either the insider financier or the entrepreneur through choosing what type of financial intermediary to associate with. We can similarly think of syndication as a case whereby the insider can credibly communicate to other members of a syndicate her information about the entrepreneur.[13] Lemma 3 then implies that for an entrepreneur with positive private benefit of continuation, more competition induces the entrepreneur to provide an experiment with a better information structure. This is reminiscent of the literature on second-sourcing (Farrell and Gallini (1988)), and is related to Bouckaert and Degryse (2004), who show that banks spontaneously provide information for strategic reasons as increasing a rival's second-period profits lowers overall competition.

In any of the above interpretations, μ captures the extent the entrepreneur is locked with the insider's investor for the later rounds of the investment. This parameter corresponds to, for example, the credit market characteristics or mutual agreement by the entrepreneur and the insider in the earlier rounds.

3.3 Relationship Formation and Information Production

We now examine MIP under the general setup with investor competition and sophistication. Not only do we show that the problem MIP persists in this general environment, but their interaction also helps us rationalize some puzzling empirical observations in the literature.

In particular, our understanding of the effect of competition on bank orientation so far has been ambiguous. The investment theory (e.g., Petersen and Rajan (1995) and Dell'Ariccia and Marquez (2004)) argues that as the credit market concentration decreases, the firms borrowing options expand, rendering banks less capable to recoup in the course of the lending relationship the initial investments in building relationship, which hinders relationship banking; the strategic theory (e.g., Boot and Thakor (2000) and Dinc (2000)) says fiercer interbank competition drives local lenders to take advantage of their competitive edge and reorient lending activities towards relational-based lending to small, local firms, which strengthens relationship banking. Others (e.g., Yafeh and Yosha (2001) and Anand and Galetovic (2006)) suggest that competition can have ambiguous effects on lending relationships, but typically predict an inverted U-shape pattern. Yet empirically Elsas (2005) and Degryse and Ongena (2007) document a U-shape relationship between likelihood of the relationship lending and the level of competition in the credit market.[14]

[13] Other syndicate members or outsider investors either observe the experiment and insider's decision, or the insider credibly conveys the interim information to them. They then bid for the rest of the issues – a case in point of the "certification" function of bank lending (e.g., Diamond (1991)).

[14] These two studies stand out because they measure relationship banking directly in terms of duration and scope of interactions, thus improve upon and complement indirect measures such as loan rate (Petersen and Rajan (1995)) or credit availability over firms' life time (Black and Strahan (2002)), for which the impact of

Proposition 2 (General MIP). *Denote the equilibrium investment function by $\mathcal{I}^{\mu,q}(X)$.*

(a) If $\varepsilon + (1-\mu)(\bar{X}(q) - I) \geq 0$, where $\bar{X}(q)$ is introduced in (4), then the entrepreneur's experimentation and investment function are as described in Lemma 2, part (a). In this case, $\mathcal{I}^{\mu,q}(X) = \mathcal{I}^{q}(X)$

(b) If $\varepsilon + (1-\mu)(\bar{X}(q) - I) < 0$, then the entrepreneur's experiment has a threshold $I - \frac{\varepsilon}{1-\mu}$, similar to that in Lemma 3, part (a). In this case, $\mathcal{I}^{q,\mu}(X) = \mathcal{I}^{\mu}(X)$

(c) Denote $\bar{K}^{\varepsilon}_{C}(\mu, q) \equiv \mu\mathbb{E}[(X - I)\mathcal{I}^{\mu,q}(X)]$ as the capacity of relationship formation for given q and μ. Then, there exists q_1 such that $\bar{K}^{\varepsilon}_{C}(\mu, q)$ is U-shaped in $[\hat{\mu}(q), 1]$ for some $\hat{\mu}(q) \in (0, 1)$, for every $q \leq q_1$. $\bar{K}^{\varepsilon}_{C}(\mu, q)$ is decreasing in μ for $q > q_1$.

(d) As $\varepsilon \to 0$, function $\bar{K}^{\varepsilon}_{C}(\mu; q)$ converges to $\bar{K}^{0}_{C}(\mu; q)$, which is increasing in μ.

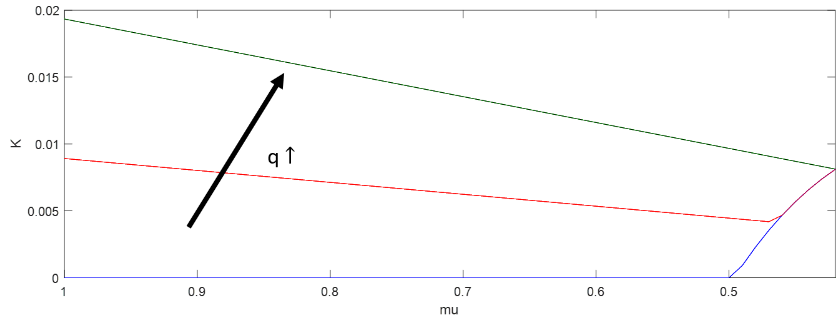
Proposition 2 shows how the level of sophistication of the lender about the project, q, and the entrepreneur's private benefit, ε, interact in shaping the investor's preference over different levels of competition μ. First of all, $\bar{K}^{\varepsilon}_{C}(\mu, q) < \mathbb{E}[\max\{X - I + \varepsilon, 0\}]$ implies some projects cannot be financed even though it could be socially efficient to form financing relationship in $t = 0$. Part (d) also confirms the general intuition in Petersen and Rajan (1995) that when the entrepreneur has aligned interests to produce informative signals about the profitability of the project, the insider prefers less interim competition.

However, as ε increases, the entrepreneur has more incentive to choose vaguer signals to increase the probability of inefficient continuation. In this case, either a sufficiently high investor sophistication or interim competition can encourage the entrepreneur to provide better information.

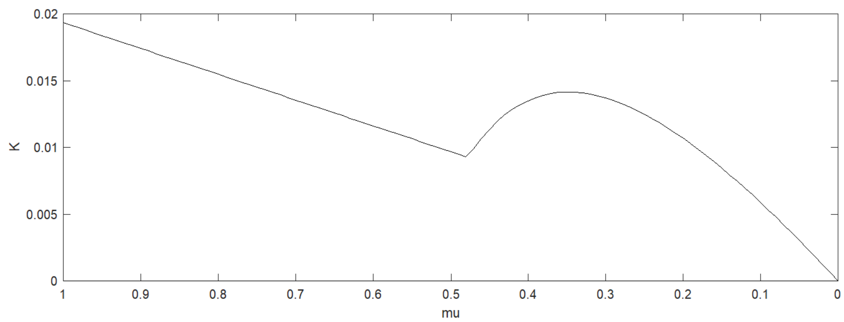
Figure 4 Panel (a) illustrates the relationship between K, the proxy for lending formation, and μ, the *inverse* measure of competition in our model. In particular, when q takes intermediate values and μ is exogenous, our model helps rationalize the findings of Elsas (2005) and Degryse and Ongena (2007).

On the one hand, for a fixed level of private benefit of continuation, lower levels of competition increases the insider's share of the surplus, and is preferred by more sophisticated investors who can get better independent information. On the other hand, higher levels of competition can encourage more efficient information production from the entrepreneur which increases total surplus, thus is preferred by the less sophisticated investors who have no other means to obtain information rent. For intermediate values of sophistication, com-

competition could be ambiguous in equilibrium (Boot and Thakor (2000)).



(a) The "U-shaped" relationship between K and μ



(b) Non-monotone relationship between K and μ

Figure 4: Illustration of the equilibrium capacity of the financing in the initial round as a function of the level of ex-post competition

petition hurts the insider's profit until it replaces the investor's independent information as her main source of interim rent, leading to the local U-shape.

Moreover, Berger, Miller, Petersen, Rajan, and Stein (2005) provide evidence consistent with small banks being better able to collect and act on soft information whereas large banks rely on hard information. To the extent that regions where large banks dominate tend to be regions with credit bureaus and alternative sources that provide hard information (higher q), our model would predict that competition would reduce relationship formation, and the opposite holds for regions with mostly small banks and mutual banks, giving an alternative explanation of what Presbitero and Zazzaro (2011) document empirically.

The model also predicts a more complex non-monotonicity: as the market becomes extremely competitive (μ gets much closer to 0), relationship formation eventually decreases in competition, as seen in Figure 4 Panel (b).

4 Contractual Solution for MIP

So far we have assumed that the entrepreneur and the insider investor use short-term contracts for financing the project. In this section, we first analyze the case that long-term dynamic contracts are feasible but the security type is exogenously given. We show that unlike the case of information monopoly (Von Thadden (1995)), these contractual measures do not fully resolve MIP. We then combine long-term contracts with endogenous security design and demonstrate that they restore social efficiency in information production and relationship financing.

Although we have used debt contracts as an example, MIP applies to all "regular" securities typically considered in the security design literature:

Definition: *A **regular security** is described by* $s(X) \in [0, X]$*, such that* $s(X)$ *and* $X - s(X)$ *are both weakly increasing.*[15]

As is standard in the literature, the security depends on the true state of the world, but is independent of the experimentation $(\mathcal{Z}, \pi)$, which is consistent with the assumption that the entrepreneur cannot commit to any experiment at the initial fundraising. With this in mind, we examine long-term contracts.

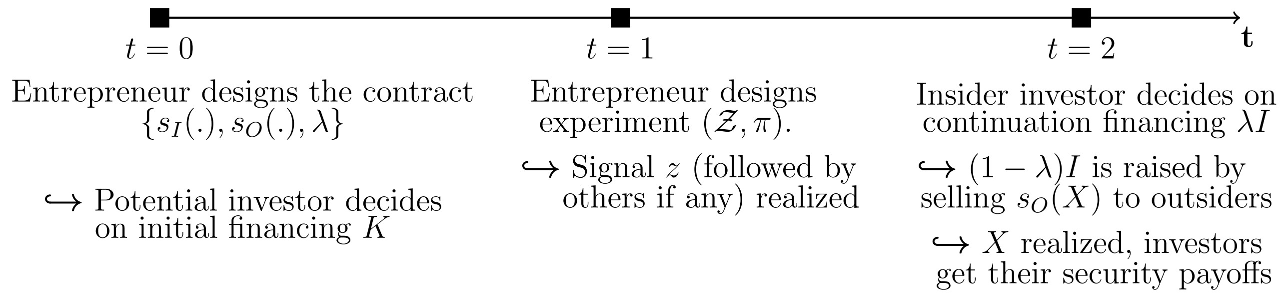
4.1 Long-term Contracts

By long-term contracts, we mean that at time 0, the entrepreneur can contract on λ, the fraction of investment I to be financed from the insider investor in the second round, and the corresponding terms $s_I(X)$. He raises $(1-\lambda)I$ by selling $s_O(X)$ to the outsider investors. Therefore, every contract can be represented by the triplet $\{s_I(.), s_O(.), \lambda\}$. Figure 5 displays the updated timing of the interactions. In particular, we assume that after the insider's decision, outsiders are conveyed the same information the insider has and bid competitively to finance $(1 - \lambda)I$.[16]

[15]See, for example, Nachman and Noe (1994); DeMarzo and Duffie (1999); DeMarzo, Kremer, and Skrzypacz (2005) and more recently, Cong (2017). If such monotonicity is violated, either the entrepreneur or the investor can be better off destroying some surplus for some state X, as pointed out in Hart and Moore (1995).

[16]For example, the insider may want to credibly share the information in the case of a financing syndicate, to do it through establishing a local credit bureau (Pagano and Jappelli (1993)). Repeated interactions of intermediaries that are typically large institutions can help facilitate credible communications about the insider's information set. In fact, in many cases the experiment itself is observable, and all we need is that

Figure 5: Timeline of the game with optimal security design.



We first show that for any given security design, it is essential to enrich a long-term contract to involve competitive outsider investors to mitigate MIP and finance innovative projects that the investor has limited private knowledge about.

Lemma 4 (Contracting with Exogenous Security Design)**.**

(a) For any given security $s(\cdot)$*, without outsider investors (*$\lambda = 1$*) and investor sophistication (*$q = \frac{1}{2}$*), the equilibrium is unique and no project is financed at* $t = 0$ *unless* $K = 0$*. The result is robust to pre-conditioning continuation on other interim events.*

(b) For every regular security $s(X)$*, there exists* $\lambda \in (0, 1)$ *and regular security* $s_O(X)$ *such that the insider investor receives positive interim rent from the contract* $\{\lambda s(\cdot), s_O(\cdot), \lambda\}$*.*

(c) For a large set of regular security designs including debt, equity, and call options, no contract in form of $\{\lambda s(\cdot), s_O(\cdot), \lambda\}$ *implements the first-best social outcome.*

Part (a) extends Lemma 1 and Proposition 1 to the case where the entrepreneur commits to a long-term contract and consequently the insider cannot bargain during the interim about the terms of the contract. This means our results do not rely on the insider investor's having information monopoly. What distinguishes the insider from outsiders in our model is that they are early investors who need to recover the initial investment through receiving interim rent, which could come from information monopoly but does not have to.

One can also imagine contracting on interim events that are related to milestones. For example, the entrepreneur can commit to reaching a pre-specified scale of customer base before seeking continued finance. As long as the insider's continuation decision still depends on the information generated from the experiment, MIP leaves no interim rent.[17] MIP is thus

the outsiders also observe the decisions of the insider.

[17]We note that contracting on milestones is different from contracting on the experiment $(\mathcal{Z}, \pi)$. Binding milestones (where the insider continues financing if conditions are met) renders endogenous information production irrelevant. It can generate interim rent but does not achieve the first-best outcome. Milestones

robust to contracting on interim events and to the timing assumptions on the determination of security terms (renegotiation).

The insider's interim rent only becomes positive with either investor sophistication or $\lambda < 1$. In fact, Lemma 4(b) shows the importance of "second-sourcing" (Farrell and Gallini (1988)) the investment I, i.e., committing to $\lambda < 1$, because the interim rent is decreasing in λ. Even the insider investor has the incentive to decrease his shares of the surplus somewhat to enable the entrepreneur to internalize the cost of inefficient investment. For most given security types, part (c) reveals that long-term contracting alone cannot restore social efficiency.

4.2 Robust Optimal Security Design and Contracting

Now we allow the entrepreneur to decide the security $s_I(X)$ for the insider investor, $s_O(X)$ for the outsider investors, and λI, the fraction of the cost of the second round of investment is financed by the insider.

Note that the entrepreneur's expected payoff from the contract (s_I, s_O, λ) is given by:

$$U^E = \int_0^1 (\varepsilon + X - s_I(X) - s_O(X) + p - (1-\lambda)I)\mathcal{I}(X)f(X)dX, \tag{6}$$

where p is the amount raised from the outsiders by selling the security $s_O(X)$, and $\mathcal{I}$ is the investment function defined earlier. Because the project cannot be continued without the insider's continuation, the outsider investors only invest when the insider continues, in order to make a profit. But competition would drive the outsiders to pay a "fair price" p, given by

$$p = \frac{1}{\mathbb{E}[\mathcal{I}(X)]}\int_0^1 s_O(X)\mathcal{I}(X)f(X)dX \tag{7}$$

Therefore, by combining (6) and (7) we get

$$U^E = \int_0^1 M(X; s_I, s_O, \lambda)\mathcal{I}(X)f(X)dX,$$

where

$$M(X; s_I, s_O, \lambda) = \varepsilon + X - s_I(X) - (1-\lambda)I, \tag{8}$$

are seldom binding in practice, because it is difficult to project milestones in early stages, and hard to enforce such contracts.

The entrepreneur then designs the contracts to solve the following maximization problem:

$$\begin{aligned} &\max_{s_I(.),s_O(.),\lambda} \mathbb{E}[M(X; s_I, s_O, \lambda)\mathcal{I}^*(X)] \\ s.t. \quad &\mathbb{E}[(s_I(X) - \lambda I)\mathcal{I}^*(X)] \geq K \quad and \quad s_I(X) + s_O(X) \leq X \quad \forall X \in [0,1], \end{aligned} \tag{9}$$

where the optimization is over the set of designs and the option to walk away from the financing relationship, and the constraints are the IC of the insider to form relationship and the entrepreneur's limited liability. $\mathcal{I}^*(\cdot)$ is the equilibrium investment function under the optimal experiment $(\mathcal{Z}^*, \pi^*)$, and is given by

$$\mathcal{I}^*(X) = \sum_{z\in\mathcal{Z}^*, y\in\{\tilde{h},\tilde{l}\}} \pi^*(z|X)\omega_q(y|X)\mathbb{I}_{\{\mathbb{E}[s_I(X)-\lambda I|z,y]\geq 0\}},$$

and the optimal experiment given the contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$ solves the optimization of information design/experimentation:

$$\begin{aligned} &\max_{(\mathcal{Z},\pi)} \int_0^1 M(X; s_I, s_O, \lambda)\mathcal{I}(X)f(X)dX \quad s.t. \quad \int_0^1 (s_O(X) - (1-\lambda)I)\mathcal{I}(X)f(X)dX \geq 0, \\ &where \quad \mathcal{I}(X) = \sum_{z\in\mathcal{Z}, y\in\{\tilde{h},\tilde{l}\}} \pi(z|X)\omega_q(y|X)\mathbb{I}_{\{\mathbb{E}[s_I(X)-\lambda I|z,y]\geq 0\}} \end{aligned}$$

Here the constraint is on the IC of outsiders to participate after observing the insider's action, which involves follow-on investment if and only if $\mathbb{E}[s_I(X) - \lambda I|z, y] \geq 0$.

Proposition 3. *(Robust Optimal Design) An optimal design exists and all optimal designs implement the first-best social outcome. In particular, the set of optimal securities that are independent of investor sophistication and entrepreneur's private bias for continuation all essentially involve the use of convertible securities:*

$$s_I(X) = \min\{\lambda I, X\}, \quad \forall\, X < I \tag{10}$$

$$\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X\geq I-\varepsilon\}}] = K \tag{11}$$

$$0 \leq s_O(X) \leq X - s_I(X), \quad \forall\, X \in [0,1] \tag{12}$$

Equation (10) requires that in the bad states of the world, the security is debt-like. In fact, the shape of the security for $X < I - \varepsilon$ is indeterminate, but in terms of payoffs are essentially unique, thus the word "essentially" when we characterize the securities. (11) requires the insider to break even ex ante and subject to that the optimal security is indeterminate in

$X \geq I$. Notice when we achieve first-best social outcome, the insider gets paid only when $X \geq I$ (payoff is zero in $[I-\varepsilon, I)$ anyway), therefore the security is independent of ϵ. Finally, (12) simply indicates limited liability of outsiders.

Figure 6 provides a concrete illustration using convertible notes with a conversion schedule for insiders and equities that are correspondingly diluted for outsiders. The dotted line indicates the threshold for continuation vs termination signals.

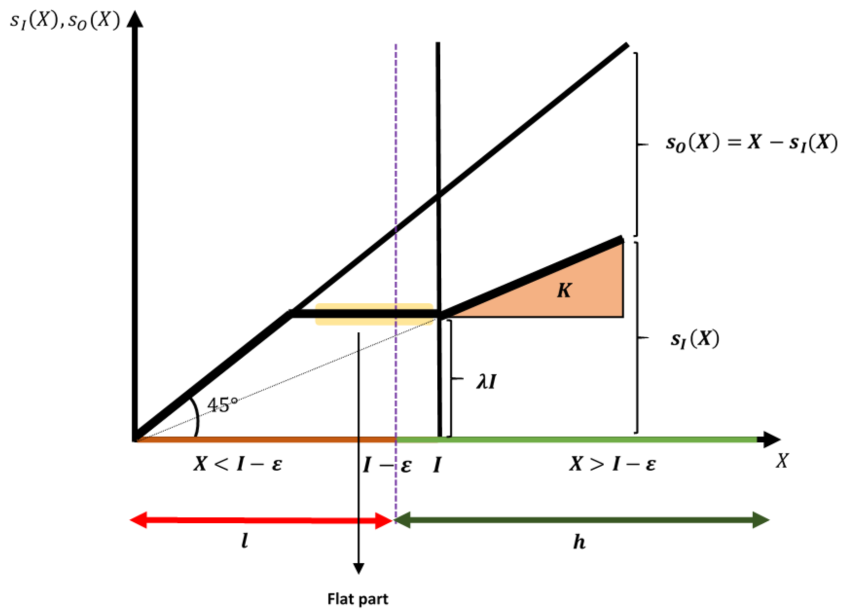


Figure 6: The optimal security under flexible security design.

Because the inefficiency lies in the continuation of bad projects, what matters is how the security allocates the downside exposure/sensitivity (when $X + \epsilon < I$) between the insider and the entrepreneur (partially through outsider investors if they are present). There are two forces at play: giving more exposure to the insider makes her less willing to continue the project, which on the margin reduces inefficient continuation (*continuation channel*); giving more exposure to the entrepreneur helps him to internalize the cost of inefficient continuation through reducing his payoff upon continuation (*payoff channel*). Note that if q is small enough, we cannot implement the first best information provision via the continuation channel. To see this, suppose q is close to $\frac{1}{2}$, the insider cannot obtain enough interim rent through the continuation channel to justify the initial investment K. Therefore a design that motivates the entrepreneur to produce information efficiently and is robust to ε and q has to exploit the direct payoff channel.

Such an optimal design should maximize the entrepreneur's downside risk relative to the insider. Therefore the entrepreneur commits to a large enough $1-\lambda$ which exposes his payoff to inefficient continuation. To make his payoff most sensitive to his information production, the entrepreneur uses debt-like contracts in the flat region in Figure 6 to give all the cash flow to the insider in the bad states of the world, enabling him to fully internalize the cost of inefficient continuation and leading to the first-best outcome. The allocation of the upside exposure only needs to ensure the insider earns enough from the second round to break even. Although this means the optimal security is indeterminate, the endogenous experimentation leads to a unique informational environment that is also socially optimal.

Our goal here is not to introduce an alternative mechanism or competing theory for the use of convertible securities or to claim the information design channel is the most dominant. Beyond deriving the optimal security under our setting, Proposition 3 indicates that earlier studies' conclusions are robust to introducing endogenous and flexible information production. Moreover, extant studies do not characterize the joint optimal securities for both early insider investors and outsiders – a phenomenon observed in real life. Our emphasis is on insider versus outsider, and should be distinguished from designing multiple classes of securities (e.g. Boot and Thakor (1993)).

4.3 Optimal Security and Contract in Practice

To map the optimal security design and long-term contracts in our model to real life practice, we can interpret the initial-round contract as warrants to the insider to purchase convertible securities in the later round. Conditional on the insider's follow-on investment, the entrepreneur finances the remaining $(1-\lambda)I$ from outsiders by issuing residual securities. Alternatively, we can view the initial round as issuing $\frac{K}{K+\lambda I}$ fraction of convertible securities to the insider, together with warrants allowing the insider to purchase the remaining fraction at a price λI in a later round. One more interpretation is that $K + \lambda I$ is the total amount of financing at the initial round, but paid out in stages, and $(1 - \lambda)I$ as separate issuance to outsiders. In all these scenarios, though round financing and stage financing in general differ (Cuny and Talmor (2005)), the effect of the optimal design remains the same.

Late-Stage Syndication

Similar to the discussion in Section 3.2, λ can also be interpreted as a commitment to later-stage syndication. While the lead investor finances λI, the remaining are financed by syndicate members that observe the interim information and are competitive. Indeed, with a few exceptions all later-stage venture capital investments are syndicated (Lerner (1994)), and partnership agreements often pre-commit venture capitalists to syndicate later-stages of investments (Sahlman (1990)).

Therefore, as we show here, syndication not only can protect the entrepreneur from ex post hold-up by investors and thereby encourage effort (Fluck, Garrison, and Myers (2006)), it also encourages more efficient information production.

Convertible Notes

Proposition 3 shows that convertible securities can implement the socially optimal information provision. To achieve this, the entrepreneur commits to the fraction of investment I that the insider can finance. λ then has practical relevance because it relates the quantity of warrants an entrepreneur should give an early investor to the initial investment K for experimentation.

Since the optimal security is indeterminate, we need to fix the type of convertible security to study the relationship between λ and K. To this end, we analyze a commonly observed security — convertible notes with conversion rate 1:I (A bond with face value λI can be converted to λ share of equity). Other optimal securities can be similarly analyzed.

Corollary 2. *Suppose* $\{s_I^{cb}(.;K), s_O^{cb}(;K), \lambda^{cb}(K)\}$ *is the contract that for a given* K*, issues convertible bond with conversion rate I:1 for the insiders and equity for the outsiders. Then,* $\lambda^{cb}(K) = \frac{K}{\mathbb{E}[\max\{X-I,0\}]}$*, and is strictly increasing in* K*.*

The corollary shows for convertible notes the positive relationship between the λ and K. It means as the cost of experimentation in the earlier stages goes up, the entrepreneur needs to commit to refinance a higher fraction of the investment with the insider, from a simple break-even condition.

4.4 Contracting under MIP

When λ is close to 1, payoff channel is not playing a big role, and we can compare MIP to traditional moral hazards of effort provision. The "efficient effort" in our case is to produce continuation only when $X + \epsilon > I$, the cost of that effort is the loss of private benefit when we terminate projects. The "action" in our setup is the experimentation, and the outcomes of the experiments correspond to noisy signals of the action.

It is a well-known result that for risk-neutral agents, the optimal security is debt (Innes (1990)). Moreover, even when we can contract on noisy signals of an agent's action, the outcome is generally not first-best (e.g., Holmstrom (1979)). Yet the optimal design is partially indeterminate in our model and restores social efficiency. This result only relies on the contractibility of actions the interim information leads to, namely continuation or termination. Our findings point to the key difference between MIP and those in conventional models.

This contrast derives from two subtle differences between our setting and the ones in Holmstrom (1979) and Innes (1990). First, the principal takes an action based on the information produced by the agent, which implies that the agent's effort can affect his final payoff through affecting the principal's continuation decision. In fact, we show in Section 5.1 that by designing a security that makes the principal's continuation decision, and thus the agent's payoff more sensitive to the agent's effort, we can also align the agent's incentives. Second, which is more important, the agent's effort in our model affects information production, but not the final output X, therefore allowing the principle to know exactly how the entrepreneur's action has affected the final payoff. In other words, upon seeing the cash flow X, everyone knows if the continuation decision is socially optimal or not, hence the entrepreneur ex ante can design the security and contract contingent on the continuation payoff to perfectly incentivize the ex-post entrepreneur to take the right action. In some sense, the noise in the entrepreneur's action — signal to continue or terminate – is orthogonal to the continuation payoff, and making the agent's payoff contingent on both fully solves the agency problem. It is also worth pointing out that unlike the literature on costly experimentation and learning (e.g., Bergemann and Hege (1998) and Hörner and Samuelson (2013)) with hidden effort and hidden information, the principal in our setting observes perfectly the signal the agent produces, which is important for attaining the first-best outcome with the optimal contract.

5 Discussion and Extension

5.1 Optimal Design of a Single Security

Suppose due to regulatory concerns, issuance costs, or high cost of communicating with outsiders in the later rounds, the entrepreneur can only use a single form of security. For example, in venture financing the right of first refusal gives its holder the contractual rights but not the obligation to purchase new security issuance before others can purchase that *same security with the same terms*; banks by regulation can only use debt contracts.

Mathematically, the entrepreneur issues security contract $s(X)$ and determines the fraction to the insiders λ, i.e., $s_I(X) = \lambda X$ and $s_O(X) = (1-\lambda)X$. Proposition 4 characterizes the equilibrium security, optimal level of commitment to outsider competition, and the optimal information production.

Proposition 4 (Optimality of Equity and Call Option)**.**
There exists $q^s \in (\frac{1}{2}, \hat{q}]$ *such that when* $q \leq q^s$*, the unique optimal security is equity and* $\lambda < 1$*; otherwise, call option is optimal, and* $\lambda = 1$ *for sufficiently small* ε*.*

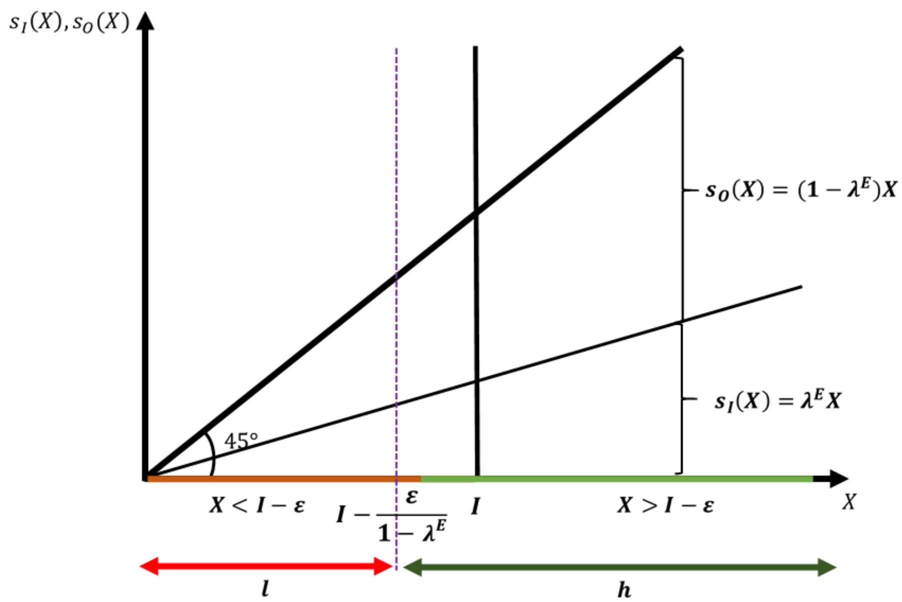


Figure 7: The optimal securities for the entrepreneur under one-security condition and without investor sophistication.

Proposition 4 shows that under low levels of sophistication, the optimal single security is equity. Recall that the entrepreneur uses the security to best commit himself to efficient interim information production. When investor sophistication is low, the payoff channel is dominant. With a single security, the entrepreneur cannot fully allocate the downside exposure to himself because both insiders and outsiders get the same security, and the outsiders are just a pass-through of interim surplus to the entrepreneur. However, by reducing λ, we naturally reduce the insider's exposure relative to the entrepreneur. Equity is thus optimal when λ is endogenous because it gives the insider the largest upside, allowing her to get

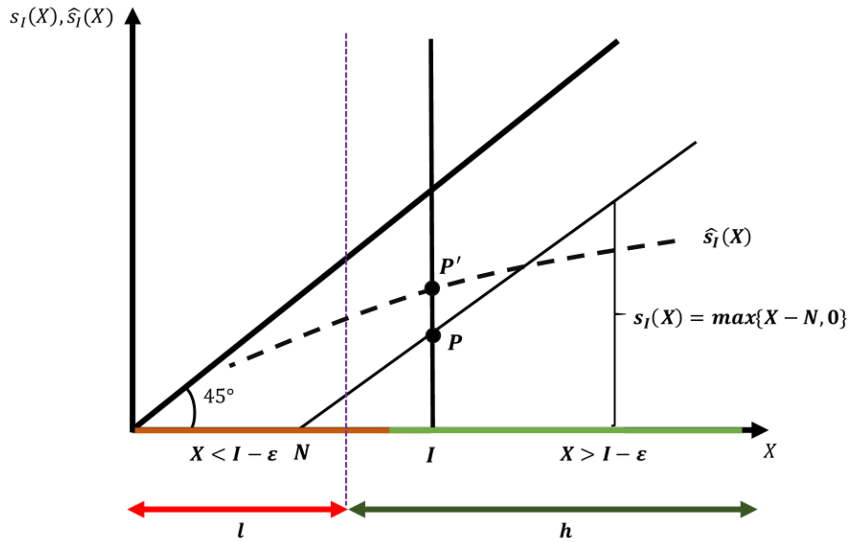


Figure 8: The optimal security under investor sophistication and without outsider investors

enough rent to cover K with the smallest λ. Figure 7 displays the optimal security for the entrepreneur.

With higher levels of investor sophistication, it is possible that the continuation channel is at work because the investor's independent information augments her ability for interim rent extraction in the second round, foiling the entrepreneur's experiment design to expropriate all interim rent. In this case, the entrepreneur wants to give all downside exposure to the insider, leading to zero payoff to the insider for low values of X. Figure 8 provides an illustration. To make the insider's continuation decision sensitive to the payoff, the entrepreneur uses the steepest security, namely call options that single crosses all other securities from below, a well-known result in security design and security-bid auctions (see for example DeMarzo, Kremer, and Skrzypacz (2005) for a discussion). Note that in this case, if implementing the first best outcome is infeasible, the entrepreneur prefers $\lambda = 1$ to expose the insider investor to downside risk as much as possible.

Corollary 3 (Optimality of Debt)**.** *If the outsiders are cash constrained or communication with them is too costly such that λ is bounded below, then debt contracts are optimal for sufficiently small values of q.*

When there is a limit on the amount of financing through the outsider investors, the entrepreneur might not be able to implement the optimal level of λ that leads to equity financing. In this case, the entrepreneur still wants the outsiders to receive the downside to help him internalize the cost of inefficient continuation the best. A debt contract allocates the highest share of the downside realizations to the outsiders, therefore it is optimal. We relegate the detailed discussion to the appendix.

5.2 Scalable Investment and Continuum Range of Actions

So far, we have assumed binary investment decision for the project. In this section, we enlarge the set of investment opportunities by considering the case that the investor can determine the scale of investment after observing the experimentation outcome.

In particular, we assume there exists weakly increasing function $r(.) : [0, 1] \to [0, 1]$, such that if the investor decides to invest αI, the project generates a stochastic cash flow $r(\alpha)X$. For the sake of consistency with the previous parts, we assume $r(0) = 0$ and $r(1) = 1$; the setting in our baseline model corresponds to the function $r(\alpha) = 0$ for $\alpha < 1$ and $r(1) = 1$. Without any interim competition, the final payoffs following the investment level α and realization of cashflow $r(\alpha)X$ are $u^E = r(\alpha)X - s(\alpha, r(\alpha)X) + r(\alpha)\varepsilon$ and $u^I = s(\alpha, r(\alpha)X) - \alpha I$ respectively, where $s(\cdot, \cdot)$ is a security payment that is generally contingent both on the project scale (or equivalently, level of investment) and the final cash flow, and private benefit of continuation depends on project scale. Note that in this setting, we assume the scaling decision is contractible. Furthermore, we assume for tractability that the investor's share from the final cashflow does not decrease as she expands the investment, i.e. $\frac{s(\alpha,r(\alpha)X)}{r(\alpha)X}$ is weakly increasing in α for every X. This assumption means that the insider demands more share of the cashflow as she puts more input into the project.

Our main results are robust under this specification of investor's action. Proposition 5 extends Proposition 1 to projects with full-benefit of scale, while the investor may receive positive interim rent for the ones with a diminishing return to scale. Furthermore, Proposition 6 extends Proposition 3 to the case of scalable investment and shows that our robust optimal design achieves the first-best outcome under general specifications and is not driven by the binary assumption of investment action.

Proposition 5 (Scalable Investment)**.**

(a) Suppose $r(\alpha) \leq \alpha$ for all $\alpha \in [0, 1]$. Then, the investor optimally chooses from $\alpha \in \{0, 1\}$. Moreover, in the absence of investor sophistication and interim competition ($q = \frac{1}{2}$ and $\lambda = 1$), the result of proposition 4(a) holds.

(b) Suppose there exists security $\tilde{s}(X)$ such that $s(\alpha, r(\alpha)X) = r(\alpha)\tilde{s}(X)$. Then, for the projects with decreasing return to scale ($r'(\alpha) > 0, r''(\alpha) < 0$) , the investor gets positive interim rent iff $\frac{1}{s(1)} < r'(0)$.

Proposition 5 shows that for projects with full-scale benefit, which includes projects with increasing and constant returns to scale, the insider investor optimally chooses one of the extreme values for the scale of the investment. We call a project is increasing return to scale (IRS)if we have $\frac{r(\alpha_2)}{\alpha_2} \geq \frac{r(\alpha_1)}{\alpha_1}$ for every $0 < \alpha_1 \leq \alpha_2$; constant and decreasing return to scale (DRS) are defined similarly. With IRS, if the investor continues the project at all, she finds full-scale investment optimal. On the other hand, if the project generates loss in expectation for her, scaling up does not help, therefore she completely terminates the project.

This fact reveals that the investor's and the entrepreneur's expected payoffs do not change upon expanding the investor's action space. The optimal experiment and the equilibrium payoffs are hence as introduced in Lemma 1. In particular, the insider investor does not get positive rent ex-post, which prevents the possibility of relationship financing ex ante.

In reality, developments of novel technologies and innovative startups often require a critical amount of investment and coordination, and exhibit constant or increasing returns to scale up to that level. In that sense, we argue that our model implications apply directly even when the scale of investment is a choice variable.

That said, we discuss the case of diminishing returns to scale in Proposition 5(b) and show that insider can retain positive interim rent even absent investor sophistication or interim competition, and investment level is interior. Nevertheless the basic economic tradeoff remains intact: suppose the investor's share of cash flow is independent of the scaling decision. If the investor makes a positive investment, she should at least break even for the marginal amount of investment. Because the production function is DRS, the investor should receive a strictly positive average return over the investment. It ensures that the investor receives strictly positive expected payoff if the entrepreneur's endogenous experimentation induces investment with positive probability. Moreover, note that the entrepreneur gets strictly positive payoff from investment. Therefore, he provides at least a good enough signal to induce investment with positive probability. Condition $\frac{I}{s(1)} < r'(0)$ provides a sufficient and necessary condition for the existence of posterior, like $f(X|z)$, that induces positive amount of investment.

In proposition 6, we revisit Lemma 4 and Proposition 3.

Proposition 6. *(Contrctual Solution under Scalable Investments)*

(a) In absence of investor sophistication or outsider investors, there is no regular security $s(\alpha, r(\alpha)X)$ that implements the first best outcome.

(b) There exists an optimal contract $\{s_I(\alpha, r(\alpha)X), s_O(\alpha, r(\alpha)X), \lambda(\alpha)\}$ that gives con-

vertible securities to insider and residual securities to outsiders, and implements the first-best outcome. If the project has full-benefit of scale, such an design is uniquely optimal.

The intuition for Proposition 6(a) resembles that for Proposition 1. If the insider receives positive interim payoff from the entrepreneur's experimentation, the entrepreneur can increase her expected payoff by designing a less informative experiment. For part (b), note that for projects with full benefit of scale, the proof follows from Proposition 3, because both the socially optimal and the investor's optimal level of investment are either the full scale or not investing at all.

To implement the first-best outcome under DRS, the security should not only encourage the entrepreneur to experiment efficiently but also incentivize the insider to efficiently scale the project. We thus make the insider's share from the investment, λ, contingent on the scale of investment, α, so that she becomes indifferent between different scales of investment. We describe the security contract in greater detail in the proof.

The key takeaway is that with DRS and a continuum of investment levels, investment decisions are no longer binary (investing I or terminating), and the investor derives partial rent from her informational monopoly. Endogenous information production still leads to reduced rent which reduces the distortion of entrepreneur incentives but potentially renders relationship financing infeasible, consistent with Lemma 1 and Proposition 1. Moreover, similar to the case of binary investments levels, there exists a contractual solution, which gives convertible security to early investors, achieves the first-best outcome, and is robust to the entrepreneur's continuation bias and investor sophistication.

5.3 General Values of q and Interval Structure of Disclosure

Throughout the paper, we have maintained the simplifying assumption that the level of the investor's sophistication is limited to $[\frac{1}{2}, \hat{q}]$. In this section, we show our main results extend to $q \in (\hat{q}, 1]$. The main complication for this case is that the entrepreneur may not use a threshold strategy for his experimentation (Lemma 5). In particular, the equilibrium experimentation has a nested-interval structure, consistent with Guo and Shmaya (2017) that finds similar structures under more general settings. For illustration, we take $\mu = 1$ in the next lemma and corollary, the case with interim competition or when $\lambda < 1$ are similar, as discussed in the proofs of earlier propositions.

Lemma 5. *For $q > \hat{q}$, there exists $q^* > \hat{q}$ such that for $q \geq q^*$, the entrepreneur optimally uses three signals $\{l, m, h\}$. The investor continues the project iff she observes $(m, \tilde{h})$, $(h, \tilde{l})$*

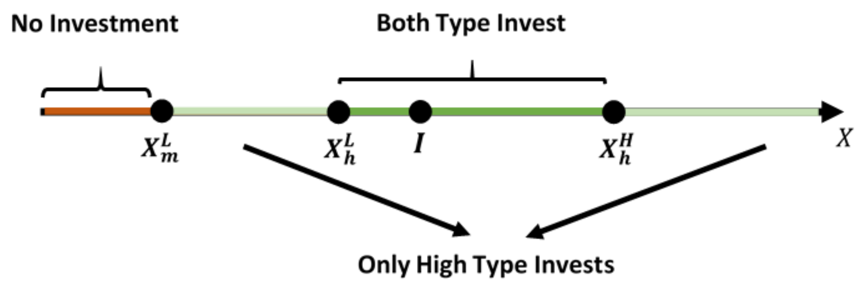


Figure 9: The nested interval structure of the entrepreneur's experimentation

or $(h, \tilde{h})$. *Specifically, there are* $X_m^L(q), X_h^L(q), X_h^H(q) \in (0,1)$ *such that the entrepreneur sends* h *in* $[X_h^L(q), X_h^H(q)]$ *and* m *in* $[X_m^L(q), X_h^L(q)) \cup (X_h^H(q), 1]$.

Corollary 4. *For* $q \geq q^*$, $\mathcal{I}^q(X) < 1$ *for a positive measure of* $X \in [I, 1]$, *implying a positive probability of inefficient termination. Moreover the investor's interim rent,* $\bar{K}_Q(q)$ *(introduced in Lemma 2), is non-monotone over the region* $(\hat{q}, 1]$.

Corollary 4 provides a counter-intuitive result that the investor's payoff is not globally increasing in investor sophistication. In fact, if the information the investor receives is very accurate, then the entrepreneur adopts a less informative experiment to increase the chance of inefficient continuations with the cost of positive probability of inefficient terminations. Similar to Proposition 5 in Kolotilin (2017), Lemma 5 highlights a "crowding-out effect" of higher levels of the investor's sophistication on the entrepreneur's information production. This effect implies that the investor might be better off by committing to ignoring some part of her independent information, which constitutes an interesting topic for future research.

In the following proposition, we show that while interim competition ($\mu < 1$) can improve the efficiency of investment decisions, it monotonically decreases the insider's interim rent for reasonably sophisticated investors, consistent with Petersen and Rajan (1995).

Proposition 7 (Impact of Competition (extended))**.**

(a) For a given security with interim renegotiation, the equilibrium investment function becomes more socially optimal as interim competition increases (μ *decreases).*

(b) There exists $q^H < 1$ *such that for* $q \in (q^H, 1)$, *the insider's interim rent is monotone in* μ. *Therefore, the insider never benefits from interim competition for very high levels of sophistication.*

Interval Structures and Disclosure Form

The interval structure exhibited in Lemma 5 also has implications on many models that leave out endogenous information production. Corporate disclosure policy in the literature is often assumed to take certain forms. For example, Diamond (1985) and Diamond and Verrecchia (1991), and more recently Goldstein and Yang (2017) use normal distributions for disclosure policy.

While it is not our main focus, our model has important implications for the form of corporate disclosure. For example, together with Szydlowski (2016), our model highlights that when the entrepreneur's (or a firm's) endogenous information production is the only source of information available to market participants, the optimal disclosure policy follows a threshold strategy. Moreover, in Section 3, we show that even if the investors receive independent signal (richer information structure), the optimal disclosure may still involve interval structures.

More generally, with endogenous information design corporate disclosures may follow interval strategies (Guo and Shmaya (2017)), leading to binary or categorical distributions instead of continuous distributions. The methodology provided in Guo and Shmaya (2017) and our paper can be used for re-examination of earlier findings that make specific distributional assumptions.

5.4 Commitment to Information Design and Partial Observation

Ex-ante Commitment to Experimentation

Commitment issues are common in the applications of information design and Bayesian persuasion in finance. Our setup relaxes this strong assumption that the entrepreneur can commit in the initial round. To see this, note that the insider has limited ability in dictating the entrepreneur's experimentation at the time of initial financing, but can observe it after becoming an insider. In other words, the entrepreneur cannot commit to an experimentation when raising K. Otherwise, the insider can extract rent (as seen in Section 3), and the problem in Section 4 becomes a joint optimization problem on security design and information disclosure for a total issuance of $K + I$, which Szydlowski (2016) addresses. He shows that both equity and debt can implement the optimal design, and the optimal security is indeterminate. In essence, the lack of commitment and contractibility of the experimentation at $t = 0$ in our setting (partially) breaks the indeterminacy in Szydlowski (2016) which has only one round of financing by fully competitive investors. An optimal security design in part contributes to the entrepreneur's commitment to efficient information production in the interim.

For illustration, suppose we use debt security to finance the project without investor sophistication, the insider investor receives less reward from high realizations. Therefore, the entrepreneur promises a larger λ to the insider in the second round to compensate for her initial investment K. The entrepreneur, less exposed to the downside (smaller $1-\lambda$), then chooses less informative signals ex post, which decreases the financing capacity in the first round. Clearly, the choice of security ex-ante affects the choice of information design ex-post.

Partial Observation of Experimentation

So far, we have assumed that the investor perfectly observes the entrepreneur's experiment and its signal realization, before making the continuation decision. In other words, we have assumed that not only the investor's monitoring technology rules out misreporting, but the investor also commits to perfectly monitor the entrepreneur's experiment, while she might be better off by randomizing between monitoring and not monitoring. In essence, this is equivalent to interim commitment of information design. We now show that while the first-best outcome becomes infeasible in the absence of full observation of the endogenous experiment, MIP and optimality of convertible securities still hold.

To incorporate the partial observation, we modify the setup to allow the possibility of the entrepreneur's misreporting and the investor's partial commitment to monitoring:

1. With given probability $\alpha \in [0,1]$, the entrepreneur can misreport the signal z, without getting caught by the investor.

2. At time $t=0$, the investor commits to verifying the experiment and its signal realization with given probability $\beta \in [0,1]$, while the entrepreneur cannot commit to truthful reporting.[18]

Note that our baseline model is the specific case of $\alpha = 0$ and $\beta = 1$, i.e., the entrepreneur cannot secretly misreport and the investor commits to monitor the experiment and its realization. In proposition 8, we revisit propositions 1 and 3.

[18]Note that if the investor monitors the experiment, she makes the continuation decision based on both the reported and the monitored signal. In this case, we allow the investor to commit to punishing the entrepreneur by not investing in case of misreporting.

Proposition 8 (Partial Observation)**.**

(a) *In the above modifications, the investor receives no interim rent in equilibrium, for any values of* $\alpha, \beta \in [0, 1]$.

(b) *For* $\alpha > 0$ *and* $\beta < 1$*, there exists no contractual solution that implements the first-best outcome. For small enough values of* α *and large enough values of* β*, the convertible securities are still optimal and robust to* ε*. Relationship financing is infeasible for large enough values of* α *and low enough values of* β*.*

Proposition 8 validates our prior knowledge that the investor's monitoring is essential to relationship financing. But part (a) shows it is insufficient. In fact, a contractual solution involving the outsiders is also reqiured to make optimal investment decisions. Furthermore, note that in contrast to costly state verification models that the investor optimally randomizes between monitoring and not monitoring, Proposition 8(b) shows that the full observation of the investor is required to implement the socially optimal outcome. The main difference is that the investor has no way to punish the entrepreneur for misreporting, as he is protected by his limited liability. Therefore, the entrepreneur always benefit from ex-post misreporting.

5.5 Security Design Right

Section 4 mainly focuses on the entrepreneur's optimal security design problem. In what follows, we show that the insider investor designs optimal security differently from what the entrepreneur does, and in a less socially efficient manner.

Proposition 9 (Security Design Right)**.** *Under both investor sophistication and unsophistication and single and flexible security design, it is more socially efficient that the entrepreneur designs the security.*

The intuition for Proposition 9 is the following: in order to raise the initial K, the entrepreneur understands he has to provide at least a minimum amount of expected cash flow to the insider investor. Therefore among all designs that generates this amount, he chooses the one that makes him commit to the most informative disclosure policy, which maximizes the total surplus. In other words, ex ante he is the residual claimant and his incentives are more aligned with a social planner.

The insider would choose a less socially optimal security design for two reasons: first, she does not consider the private benefit of the entrepreneur; second, she not only weighs the total expected cash-flow, but also cares for her share from the output which distorts

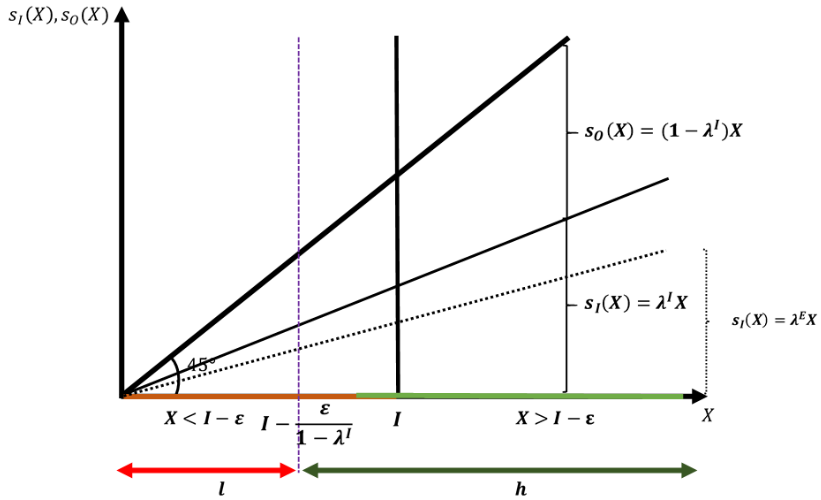


Figure 10: The optimal security for the insider investor under single security design.

the entrepreneur's incentives. A comparison between the two panels in Figures 10 and 7 graphically reveals that the insider's design of λ_I leads to a continuation decision further away from what is socially optimal.

5.6 Conventional Effort Distortion

For simplicity, we have emphasized the entrepreneur's role as an information provider, whereas earlier studies concern entrepreneur's costly effort to improve project cash flows. In this section, we discuss how these two actions interact.

For modeling effort, we assume the entrepreneur can choose from a set of conditional distributions $f(X|e)$, where $e \in \mathcal{E}$ is the set of available levels of effort. Function $c(\cdot) : \mathcal{E} \to \mathcal{R}$ shows the cost associated with each level of effort. First, suppose $\mathcal{I}(X; e)$ is the equilibrium investment function when effort level e is chosen. Then, $e^* \in \mathcal{E}$ is *constrained first-best* if it solves the following maximization problem:

$$e^* \in \quad argmax \ \mathbb{E}[(X - I + \varepsilon)\mathcal{I}(X; e)|e] - c(e) \tag{13}$$

It is straight-forward to show that absent investor sophistication and interim competition, there is no effort distortion: given the equilibrium information structure for each level of effort, the entrepreneur chooses the one that is socially preferred. Neither the insider's information monopoly nor the moral hazard of information production distorts entrepreneurial effort. The reason is when the insider investor gets no rent, the entrepreneur fully internalizes the benefit and the cost of effort. Naturally, in presence of a sophisticated investor and interim competition, the investor may get positive interim rent that distorts effort provision, but still to a lesser extent compared to the case where information production is exogenous and the insider enjoys full monopoly rent.

6 Conclusion

Motivated by the dynamic financing of innovative projects by relationship arms-length investors, we study a Bayesian persuasion game with hold-ups and contingent transfers. We show that the entrepreneur (sender)'s endogenous information production typically follows threshold strategies, and its associated moral hazard can eliminate interim rent of the insider investor (receiver) despite the latter's informational monopoly or bargaining power. The non-contractibility of information production hinders the relationship formation in general. We then show that investor sophistication (receiver's type) and interim competition can mitigate the problem, and they interact to produce patterns on relationship financing documented in the empirical literature, including the U-shaped link between relationship lending and competition. Importantly, we derive optimal sequential securities to solve the moral hazard of information production: the entrepreneur contracts with investors in the initial round to allow them to purchase convertible securities in a later round, and issue residual claims to competitive outsiders later. The optimal security choice and contracting terms are robust to the entrepreneur's continuation bias, investor sophistication, and scalable investment, and can be applied to similar hold-up problems in persuasion games with contingent transfers, even beyond relationship lending and venture capital.

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Appendix: Proofs and Extended Discussions

A1. A Technical Lemma

The entrepreneur endogenously designs the experiment to maximize his payoff, subject to satisfying the insider investors' second round participation constraint. With finite state space, the signal space as the range of a deterministic mapping from the state space is necessarily finite. Consequently, we can apply the method of Lagrange multipliers directly. But alas, we are dealing with infinite dimensional state space, and in unrestricted signal generation space, we cannot always apply the method of Lagrange multipliers.

That said, the optimal experimentation function is given by the characteristic function of a countable sup-level set of payoff densities, for some cutoff value "multiplier". In other words, the experimentation we look at are conditional probabilities and therefore their space is a Banach space. With this insight, we first prove the following mathematical lemma that allows us to use the method of Lagrange multipliers in the proofs of our lemmas and propositions (see also Ito (2016) for an abstract generalization).

Lemma (A1). *Suppose $w_i(x), m_i(x) : [0,1] \to \Re$ $(1 \leq i \leq N)$ are continuous and bounded functions. Suppose the following maximization problem has a solution:*

$$\begin{aligned} &\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx \\ s.t. \quad &\int_0^1 m_i(x)\alpha_i(x)dx \geq 0 \quad \forall\ 1 \leq i \leq N, \quad \textit{and} \quad \sum_{i=1}^N \alpha_i(x) \leq 1 \quad \forall x \in [0,1], \end{aligned} \tag{14}$$

where $\mathcal{A}$ is the set of all measurable functions over $[0,1]$ that take value from $[0,1]$. Then, there exist non-negative real numbers $\{\mu_i\}_{i=1}^N$, such that the solution to (14) is a solution the following maximization problem:

$$\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N (w_i(x) + \mu_i m_i(x))\alpha_i(x)dx \quad s.t. \quad \sum_{i=1}^N \alpha_i(x) \leq 1 \quad \forall x \in [0,1] \tag{15}$$

Proof. Let $\tilde{\mathcal{A}}^N$ be the set of all $N-$tuples of functions $(\alpha_1(.), \dots, \alpha_N(.))$ in $\mathcal{A}$ that satisfy $\sum_{i=1}^N \alpha_i(x) \leq 1$.

Since all functions are bounded and measurable, then it is easy to check that $\tilde{\mathcal{A}}^N$ constitutes a closed set in ${\mathcal{L}^1}^N$. Therefore, the following maximization problem is well-defined.

$$\max_{(\alpha_1(.),\dots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx \quad s.t. \quad \int_0^1 m_i(x)\alpha_i(x)dx \geq 0 \quad \forall\ 1 \leq i \leq N \tag{16}$$

Suppose $a^* \in \tilde{A}^N$ is the solution to the problems (14) and (16). It is easy to see that Slater condition, and correspondingly, strong duality holds. Therefore, there exists a vector of non-negative real numbers $\{\mu_i\}_{i=1}^N$ such that a^* solves the following maximization problem as well:

$$\max_{(\alpha_1(.),\dots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx + \sum_{i=1}^N \mu_i \int_0^1 m_i(x)\alpha_i(x)dx \tag{17}$$

Note that (15) is equivalent to (17). It completes the proof. □

In fact, for most Bayesian persuasion settings studied in the literature, the signal generation function corresponds to mapping to probability space that is bounded, and therefore lies in a Banach space. This allows us to apply the approach in Bergemann and Morris (2017) beyond finite-state-space settings.

A2. Proof of Lemma 1

We prove the lemma for "regular" securities (defined in Section 4) which require $s(X)$ and $X - s(X)$ to be weakly increasing. The case of debt contracts with the insider's full bargaining power (as is the case for Lemma 1) is equivalent to having $s(X) = X$, which is a special case of the regular securities. Furthermore, we assume $s(X) \geq I$ with positive probability. If this condition does not hold, the project would be terminated with probability one for every experiment.

First, we show that two signals are enough to implement the optimal information design. To do this, we show that for every experiment $(\mathcal{Z}, \pi)$, there exists an experiment with two signals like $(\{z_1, z_2\}, \tilde{\pi})$ that implements the same investment function, i.e. $\mathcal{I}(X) = \tilde{\mathcal{I}}(X)$ a.s, where $\mathcal{I}(\cdot)$ and $\tilde{\mathcal{I}}(\cdot)$ are the corresponding investment functions. Given this, there is no loss of generality to only examine optimal experiments with two signals.

For experiment $(\mathcal{Z}, \pi)$, define $\mathcal{Z}^+ = \{z \in \mathcal{Z} | \mathbb{E}[s(X)|z] \geq I\}$ and $\mathcal{Z}^- = \{z \in \mathcal{Z} | \mathbb{E}[s(X)|z] < I\}$ to be the set of signals that induce investment and termination respectively. Note that for every $z \in \mathcal{Z}^+$ we have $\mathbb{E}[s(X) - I|z] \geq 0 \iff \int_0^1 (s(X) - I)\pi(z|X)f(X) \geq 0$. Integrating the recent relation over all $z \in \mathcal{Z}^+$ gives:

$$\int_0^1 (s(X) - I)[\int_{Z^+} \pi(z|X)dz]f(X) \geq 0$$

Similarly, we have:

$$\int_0^1 (s(X) - I)[\int_{Z^-} \pi(z|X)dz]f(X) < 0$$

Now define a new experiment $(\{z_1, z_2\}, \tilde{\pi})$ as follows:

$$\tilde{\pi}(z_1|X) = \int_{Z^+} \pi(z|X)dz \qquad \tilde{\pi}(z_2|X) = \int_{Z^-} \pi(z|X)dz$$

It is clear that the investor invests if she receives z_1 and she does not if z_2 is received. Therefore, for the investment functions we have the following

$$\mathcal{I}(X) = \tilde{\pi}(z_1|X) = \int_{Z^+} \pi(z|X)dz = \tilde{\mathcal{I}}(X) \tag{18}$$

Note that the equilibrium payoff of the entrepreneur from an experiment with investment function $\mathcal{I}_\pi(\cdot)$ is given by:

$$U^E(\mathcal{Z}, \pi) = \mathbb{E}[(X - s(X) + \varepsilon)\mathcal{I}_\pi(X)]$$

Therefore the entrepreneur receives the same expected payoff from two experiments if their investment functions are equal, almost surely. In other words, we optimize over all measurable functions $\tilde{\pi}(z_1|X) \in [0, 1]$:

$$\max_{\tilde{\pi}(z_1|X)} \int_0^1 [\varepsilon + X - s(X)]\tilde{\pi}(z_1|X)f(X)dX \tag{19}$$

$$s.t. \int_0^1 (s(X) - I)\tilde{\pi}(z_1|X)f(X)dX \geq 0, \quad and \quad \tilde{\pi}(z_1|X) \in [0, 1]$$

Note that $s(X) \geq I$, with positive probability and the set of measurable functions satisfying the constraint in (19) is a closed and bounded subset of $\mathcal{L}^1$. Therefore, (19) has a solution. Using Lemma A1 with

$N=1$, let λ be the corresponding multiplier μ. Then the optimal experiment $\pi^*(z_1|X)$ is the solution to:

$$\max_{\tilde{\pi}(z_1|X)} [\varepsilon + X - s(X) + \lambda(s(X) - I)]\tilde{\pi}(z_1|X)f(X) \tag{20}$$

Also note that the term in the first bracket is strictly increasing in X. Therefore, the optimal experiment implements a threshold strategy, i.e. there exists $\bar{X}_s \in [0,1]$, such that $\tilde{\mathcal{I}}(X) = \mathbb{I}_{\{X \geq \bar{X}_s\}}$. The threshold satisfies the following:

$$\varepsilon + \bar{X}_s - s(\bar{X}_s) + \lambda(s(\bar{X}_s) - I) = 0$$

Note that $\lambda > 0$, since otherwise $\pi^*(z_1|X) = 1$ for all $X \in [0,1]$, yet $\mathbb{E}[s(X)] \leq \mathbb{E}[X] < I$ — a contradiction. Therefore, the constraint in (19) is binding. It implies $\int_{\bar{X}_s}^1 (s(X) - I)f(X)dX = 0$. For the special case of $s(X) = X$, (3) implies that the threshold is $\bar{X}$.

Therefore, the optimal two-signal experiment and its investment function then can be characterized as:

$$\pi^*(X) = \begin{cases} h & X \in [\bar{X}_s, 1] \\ l & X \in [0, \bar{X}_s] \end{cases} \qquad \mathcal{I}^*(X) = \begin{cases} 1 & X \in [\bar{X}_s, 1] \\ 0 & X \in [0, \bar{X}_s] \end{cases} \tag{21}$$

The expected payoffs from the optimal experiments are:

$$\begin{aligned} U^E(\{h,l\}, \pi^*) &= \mathbb{E}[(X - s(X) + \varepsilon)\mathcal{I}^*(X)] = \int_{\bar{X}_s}^1 (X - I + \varepsilon)f(X)dX = \varepsilon(1 - F(\bar{X}_s)) \\ U^I(\{h,l\}, \pi^*) &= -K + \mathbb{E}[(s(X) - I)\mathcal{I}^*(X)] = -K + \int_{\bar{X}_s}^1 (s(X) - I)f(X)dX = -K \end{aligned} \tag{22}$$

where the last equality follows from $\int_{\bar{X}_s}^1 (s(X) - I)f(X)dX = 0$. $s(X) = X$ is just a special case.

Next, we show the optimization problem (19) has a unique solution. define $i(.) : \Delta([0,1]) \to [0,1]$ as the investment function that the investor continues the project with probability one when she weakly prefers continuation. So far, we showed that $(\{h,l\}, \pi^*)$ and $i(.)$ constitute an equilibrium.

To rule out mixed-strategy equilibrium, we show that there exists no Nash Equilibrium in which the investor terminates the project with positive probability when she is indifferent between continuation and termination. Suppose the contrary that there exists a Nash Equilibrium in which the entrepreneur uses experiment $(\mathcal{Z}', \pi')$ and the investor uses the investment function $i'(.) : \Delta([0,1]) \to [0,1]$. Note that for every experiment and every signal $z \in \mathcal{Z}'$: $\mathbb{E}[s(X) - I|z] > 0 \Rightarrow i'(z) = 1$.

Define $\pi_c(.)$ as an experiment that sends a high signal for $X \geq c$. Note that the entrepreneur's payoff from $(\{h,l\}, \pi_c)$ for $c > \bar{X}$ is $\varepsilon(1 - F(c))$, as the investor receives positive expected payoff from continuation following a high signal. Note that the payoff converges to the entrepreneur's payoff from the optimal experiment as c converges to $\bar{X}_s$ from above. Therefore, the entrepreneur's payoff from $(\mathcal{Z}', \pi')$ should be as high as the optimal experiment.

Note that there exists a signal in $\mathcal{Z}'$ that makes the investor indifferent, otherwise we can simply construct another experiment that gives higher expected payoff to the entrepreneur, when the investor plays $i(.)$. Moreover, by assumption, in the new equilibrium, the investor terminates with positive probability when she is indifferent. Therefore, there exists a signal $z \in \mathcal{Z}'$ such that $i'(z) < i(z)$. It implies when the insider plays $i(.)$, the entrepreneur strictly prefers $(\mathcal{Z}', \pi')$ to $(\{h,l\}, \pi^*)$, which is a contradiction with the optimality of π^*. Therefore, there is no other Nash Equilibrium.

Finally, we have shown earlier that all optimal experiments share the same investment functions induced by the two-signal experiment, and (21) implements the same payoffs as in all such experiments. The equilibrium is essentially unique.

A3. Proof of Corollary 1

According to (22), the total surplus in equilibrium is $\varepsilon(1 - F(\bar{X})) - K$. Therefore, it is socially optimal to start the relationship financing iff $\varepsilon(1 - F(\bar{X})) - K \geq 0$.

A4. Proof of Proposition 1

As it is shown in Lemma 1, the investor gets zero interim rent, and is unable to recover any positive initial payment K made at $t = 0$. Uniqueness is shown in the proof of Lemma 1.

A5. Proof of Lemma 2

Similar to the proof of Lemma 1, we prove the lemma for regular securities $s(X)$. First we note that if $\mathbb{E}[s(X) - I|X > I] \leq 0$, then $\bar{X}_s \geq I$, where $\bar{X}_s$ is the threshold introduced in the proof of Lemma 1. In this case, the investor's signal y is not used and the equilibrium experiments, investment functions and payoffs are the same as the ones provided in Lemma 1. As such, the remaining proof is centered on the case $\mathbb{E}[s(X) - I|X > I] > 0$ for simplicity in exposition.

Proof for Part (a)

Similar to the proof of Lemma 1, first we show that for every experiment, there is another experiment with bounded number of signals that gives the entrepreneur the same expected payoff. It helps us to prove the existence of optimal experiments and then characterize them. We refer to the private information (broadly defined) the insider has as *investor type*.

Lemma (A2). *Denote T as the set of investor type and A the set of actions. As long as A and T are finite, for every experiment $(\mathcal{Z}, \pi)$, there exists experiment $(\mathcal{Z}', \pi')$ such that $|\mathcal{Z}'| \leq |A|^{|T|}$ and $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$.*

Proof. For every pure strategy of the investor, such as $\mathfrak{a}(\cdot) : T \to A$, define $\mathcal{Z}(\mathfrak{a})$ as the set of signals in $\mathcal{Z}$, such as z, that the investor chooses $\mathfrak{a}(t)$ when her type is t and she receives z. Note that if $\mathcal{Z}(\mathfrak{a})$ is non-empty, then:

$$\begin{aligned} &\mathbb{E}[u^I(\mathfrak{a}(t), X)|t, z] \geq \mathbb{E}[u^I(a', X)|t, z] \quad \forall t \in T, z \in \mathcal{Z}(\mathfrak{a}) \\ \Rightarrow &\mathbb{E}[u^I(\mathfrak{a}(t), X)|t, \mathcal{Z}(\mathfrak{a})] \geq \mathbb{E}[u^I(a', X)|t, \mathcal{Z}(\mathfrak{a})] \quad \forall t \in T \end{aligned}$$

where $u^I(a, X)$ is the final payoff of the investor from the action a in state X. Now define the experiment $(\mathcal{Z}', \pi')$ as follows: $\mathcal{Z}' = \{z_{\mathfrak{a}}\}_{\mathfrak{a} \in A^T}$, for all $\mathfrak{a}$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. Moreover, define

$$\pi'(z_{\mathfrak{a}}|X) = \sum_{z \in \mathcal{Z}(\mathfrak{a})} \pi(z|X) \quad \text{if } \mathcal{Z}(\mathfrak{a}, t) \text{ is non-empty}$$

Note that by definition, $z_{\mathfrak{a}}$ is the signal in $\mathcal{Z}'$ that induces the strategy profile $\mathfrak{a}(t)$. We only need to show that $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$. To see this,

$$U^E(\mathcal{Z}', \pi') = \int_0^1 \sum_{t \in T} \sum_{z_{\mathfrak{a}} \in \mathcal{Z}'} u^E(\mathfrak{a}(t), X) \pi'(z_{\mathfrak{a}}|X) g(t|X) f(X) dX$$
$$= \int_0^1 \sum_{t \in T} \left[\int_{z \in \mathcal{Z}} u^E(a(z, t), X) \pi(z|X) dz \right] g(t|X) f(X) dX = U^E(\mathcal{Z}, \pi)$$

where $a(z,t)$ is the investor's action for type t upon receiving signal z. This completes the proof. $\square$

Building from the above result, the next lemma proves the existence of an optimal experiment and characterizes it.

Lemma (A3). ***(a)*** *Suppose the investor's action is binary ($A = \{0,1\}$) and investor types are ordered by $T = \{t_1, t_2, \ldots, t_{|T|}\}$ such that posteriors $u^I(1,X) - u^I(0,X)|t_i$ are ranked by first-order stochastic dominance. Then for every experiment, there is an experiment that implements the same expected payoffs and uses at most $|T|+1$ signals.* ***(b)*** *Under these conditions, an optimal experiment exists.*

Proof.

Proof of Part (a)

Note that under single-property condition, $a(z,t)$ is weakly increasing in t. Therefore, there are at most $|T|+1$ functions like $\mathfrak{a} : T \to A$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. With a similar argument as the one in the proof of the lemma, one can construct an experiment with at most $|T|+1$ signals that implements the same expected payoffs.

Proof of Part (b)

To show the existence of the optimal experiment, we only need to look at experiments with at most $|T|+1$ signals. In particular, we only need to show that the conditional distributions $\pi(z|X)$, $z \in \mathcal{Z}$, constitute a closed bounded set in ${\mathcal{L}^1}^{|T|+1}$.

For an experiment $(\mathcal{Z}, \pi)$, we can assume it has at most one signal, like $z_i \in \mathcal{Z}$, such that the entrepreneur chooses $a = 1$ only if her types is t_i or above. $z_{|T|+1}$ is the signal following which no types would invest. Therefore, the part (a) shows that every experiment implements the same expected payoffs with an experiment with following conditions:

$$\begin{aligned} &\int_0^1 (u^I(1,X) - u^I(0,X))\pi(z_i|X)g(t_j|X)f(X) \geq 0 \quad \text{ iff } j \geq i, \; \forall 1 \leq i \leq |T|+1 \\ &\sum_{i=1}^{|T|+1} \pi(z_i|X) = 1 \quad \forall X \in [0,1], \quad \textit{and} \quad \pi(z_i|X) \geq 0 \quad \forall X \in [0,1], z_i \in \mathcal{Z} \end{aligned} \tag{23}$$

It is easy to check that the set of experiments satisfying (23) is closed and bounded. Therefore, an optimal experiment exists.

$\square$

As a result of the lemma and the corollary, the optimal experiment exists and has at most three signals. An exhaustive list of candidate signal ranges for such an optimal experiment is $\{m,l\}$, $\{h,l\}$ or $\{m,h,l\}$, where the investor only invests if she receives $(m,\tilde{h})$, $(h,\tilde{l})$ or $(h,\tilde{h})$. We show that for $q \leq \hat{q}$, the optimal experiment is essentially unique and it is of the second form.

Lemma (A4). *No two-signal experiment in the form of $(\{l,m\},\pi)$, where the investor invests iff she receives $(m,\tilde{h})$, is optimal.*

Proof. Suppose the contrary and suppose that there exists an optimal experiment like $(\{m,l\},\pi^*_M)$. Therefore, π^*_M solves the following maximization problem:

$$\begin{aligned} \max_{\pi(m|X)} \; &(1-q)\int_0^I (\varepsilon + X - s(X))\pi(m|X)f(X)dX + q\int_I^1 (\varepsilon + X - s(X))\pi(m|X)f(X)dX \\ s.t. \quad &(1-q)\int_0^I (s(X) - I)\pi(m|X)f(X)dX + q\int_I^1 (s(X) - I)\pi(m|X)f(X)dX \geq 0 \end{aligned}$$

$$\pi(m|X) \in [0,1] \qquad \forall X \in [0,1]$$

Let κ be the multiplier corresponding to the constraint. π^*_M then maximizes the following objective function, given the constraint $\pi^*_M(m|.) \in [0,1]$.

$$\max_{\pi(m|X)} (1-q)\int_0^I [\varepsilon + X - s(X) + \kappa(s(X) - I)]\pi(m|X)f(X)dX$$
$$+q\int_I^1 [\varepsilon + X - s(X) + \kappa(s(X) - I)]\pi(m|X)f(X)dX$$

Since both $X - s(X)$ and $s(X) - I$ are weakly increasing functions, there is a threshold value $X_m \in [0,1]$ such that for all $X \geq X_m$, the expression in the bracket is non-negative. According to lemma A1, the optimal experiment among those that only implement signals m and l has a threshold scheme, where the threshold X_m satisfies the following:

$$(1-q)\int_{X_m}^I (s(X) - I)f(X)dX + q\int_I^1 (s(X) - I)f(X)dX = 0$$

In this case, the expected utility of the entrepreneur from $(\{m,l\}, \pi^*_M)$ is:

$$U^E(\{m,l\}, \pi^*_M) = (1-q)\int_{X_m}^I (\varepsilon + X - s(X))f(X)dX + q\int_I^1 (\varepsilon + X - s(X))f(X)dX \tag{24}$$

By comparing the recent equality with (4), it is easy to see that $X_m < \bar{X}(q)$. Now, we show how the entrepreneur can improve upon π^*_M by introducing signal h (a signal that induces investment regardless of the signal the investor receives). To show this, we consider two cases:

- $s(I) < I$: In this case, we can find a subset $A \subset [I,1]$ such that $\int_A (s(X) - I)f(X)dX = 0$. Then an experiment that sends h for the members of A ($\pi(h|X) = 1$ iff $X \in A$) and sends m for $[X_m, 1] \setminus A$ implements higher payoff for the entrepreneur by $(1-q)\int_A (\varepsilon + X - s(X))f(X)dX$.
- $s(I) = I$: Since $X - s(X)$ is a weakly increasing function, it implies that $s(X) = X$ for all $X \leq I$. Consider small positive values $\eta_1, \eta_2, \eta_3 \geq 0$ that satisfy the following:

$$(1-q)\int_{X_m+\eta_1}^{I-\eta_2} (s(X) - I)f(X)dX + q\int_{I+\eta_3}^1 (s(X) - I)f(X)dX \geq 0$$

$$q\int_{I-\eta_2}^{I} (s(X) - I)f(X)dX + (1-q)\int_{I+\eta_3}^1 (s(X) - I)f(X)dX \geq 0$$

and introduce the following alternative experiment $(\{l,m,h\}, \tilde{\pi}_M)$:

$$\tilde{\pi}_M(h|X) = \begin{cases} 1 & X \in [I-\eta_2, I+\eta_3) \\ 0 & X \in [0, I-\eta_2) \cup [I+\eta_3, 1] \end{cases} \qquad \tilde{\pi}_M(m|X) = \begin{cases} 1 & X \in [X_m+\eta_1, I-\eta_2) \cup [I+\eta_3, 1] \\ 0 & X \in [0, X_m+\eta_1) \cup [I-\eta_2, I+\eta_3) \end{cases}$$

It is easy to verify that the experiment $\tilde{\pi}_M$ is designed in a way that the investor invests iff she receives one of $(m,\tilde{l})$, $(h,\tilde{l})$ or $(h,\tilde{h})$. Now, the difference in expected payoffs for the entrepreneur is given by:

$$U^E(\{m,l\},\tilde{\pi}_M) - U^E(\{m,l\},\pi^*_M) = -(1-q)\int_{X_m}^{X_m+\eta_1}(\varepsilon + X - s(X))f(X)dX$$
$$+q\int_{I-\eta_2}^{I}(\varepsilon + X - s(X))f(X)dX + (1-q)\int_{I}^{I+\eta_3}(\varepsilon + X - s(X))f(X)dX$$

Now consider the contrary, that the introduced experiment is an optimal experiment. Then $\eta_1^* = \eta_2^* = \eta_3^* = 0$ should satisfy the first order conditions for the following maximization problem:

$$\max_{\eta_1,\eta_2,\eta_3} \quad -(1-q)\int_{X_m}^{X_m+\eta_1}(\varepsilon + X - s(X))f(X)dX + q\int_{I-\eta_2}^{I}(\varepsilon + X - s(X))f(X)dX$$
$$+(1-q)\int_{I}^{I+\eta_3}(\varepsilon + X - s(X))f(X)dX$$

$$s.t. \quad \eta_1,\eta_2 \geq 0, \qquad q\int_{I-\eta_2}^{I}(s(X)-I)f(X)dX + (1-q)\int_{I}^{I+\eta_3}(s(X)-I)f(X)dX \geq 0, \quad and$$
$$(1-q)\left[\int_{X_m}^{X_m+\eta_1}(s(X)-I)f(X)dX + \int_{I-\eta_2}^{I}(s(X)-I)f(X)dX\right] + q\int_{I}^{I+\eta_3}(s(X)-I)f(X)dX \leq 0$$

Let κ_1 and κ_2 be the multipliers for the first constraints, respectively. Since $s(I) = I$, the FOC for η_2 is positive at $\eta_2 = 0$: $[\eta_2]|_{\eta_2=0} : q\varepsilon f(I)$.

Similarly, the FOC for η_3 is positive at $\eta_3 = 0$. Therefore, the optimal values are non-zero. It shows that the experiment $(\{m,l\},\pi^*_M)$ cannot be optimal for any $q \in (\frac{1}{2}, 1)$.

□

Lemma (A5). *No three-signal experiment in the form of $(\{l,m,h\},\pi)$, whereby signals m and h are both sent with positive probability, is optimal.*

Proof. First note we are considering the parameter range $q \in (\frac{1}{2}, \hat{q}]$. Suppose the contrary and there exists a three-signal optimal experiment like $(\{l,m,h\},\pi^*_{HM})$, where the investor invests iff she receives one of $(m,\tilde{l})$, $(h,\tilde{l})$ and $(h,\tilde{h})$. Then $\pi^*_{HM}(m|X)$ and $\pi^*_{HM}(h|X)$ solve the following optimization problem.

$$\max_{\pi(h|X),\pi(m|X)} \int_0^I (\varepsilon + X - s(X))(\pi(h|X) + (1-q)\pi(m|X))f(X)dX$$
$$+\int_I^1 (\varepsilon + X - s(X))(\pi(h|X) + q\pi(m|X))f(X)dX \tag{25}$$

$$s.t. \quad q\int_0^I (s(X)-I)\pi(h|X)f(X)dX + (1-q)\int_I^1 (s(X)-I)\pi(h|X)f(X)dX \geq 0$$
$$(1-q)\int_0^I (s(X)-I)\pi(m|X)f(X)dX + q\int_I^1 (s(X)-I)\pi(m|X)f(X)dX \geq 0$$
$$\pi(h|X),\pi(m|X) \in [0,1]$$

Let λ^h and λ^m be the multipliers for the first two restrictions, respectively. Define $c_m(X)$ and $c_h(X)$ as follows:

$$c_h(X) = \begin{cases} \varepsilon + X - s(X) + q\lambda^h(s(X)-I) & 0 \leq X < I \\ \varepsilon + X - s(X) + (1-q)\lambda^h(s(X)-I) & I \leq X \leq 1 \end{cases}$$

$$c_m(X) = \begin{cases} (1-q)(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & 0 \le X < I \\ q(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & I \le X \le 1 \end{cases}$$

Then $\pi(h|X)$ and $\pi(m|X)$ solves the following optimization problem subject to $0 \le \pi(h|X), \pi(m|X) \le 1$

$$\max_{\pi(h|X),\pi(m|X)\in[0,1]} \int_0^1 [c_h(X)\pi(h|X) + c_m(X)\pi(m|X)]f(X)dX \tag{26}$$

Now, we appeal to lemma A1. Note that the optimization problem (26) is linear in $\pi(h|X)$ and $\pi(m|X)$. Moreover, it is easy to see that their multipliers are equal at most in a measure-zero subset of $[0, 1]$. Therefore, $\pi(h|X), \pi(m|X) \in \{0, 1\}$ *a.s.*. Therefore, there are two subsets $M, H \in [0, 1]$ such that the signals m and h are sent for the members in M and H, respectively. Moreover, define $M^1 = M \cap [0, I)$, $M^2 = [I, 1] \cap [I, 1]$ and define H^1 and H^2, correspondingly.

If M_1 is empty, then signal m is just sent for a subset of $X \in [I, 1]$. In this case, the investor invests even if she receives $(m, \tilde{l})$, which is in contrast with the definition of signal m. Therefore, suppose M_1 is non-empty. For $X \in M_1$ we have $c_m(X) \ge \max\{c_h(X), 0\}$. Rearranging the expressions of $c_h(X)$ and $c_m(X)$, we have

$$\frac{q\lambda^h - (1-q)\lambda^m}{q} \ge \frac{\varepsilon + X - s(X)}{I - s(X)} \ge \lambda^m \Rightarrow q\lambda^h \ge \lambda^m \tag{27}$$

Moreover, note that if M_2 is empty, then the investor does not ever invest when she receives m, which is a contradiction with the definition of m. Therefore, M_2 is not empty and there exists $X \in M_2$. For X, we have:

$$\begin{aligned} c_m(X) \ge c_h(X) \quad &\Rightarrow \quad (q\lambda^m - (1-q)\lambda^h)(s(X) - I) \ge (1-q)(\varepsilon + X - s(X)) \\ &\Rightarrow \quad q\lambda^m - (1-q)\lambda^h > 0 \end{aligned} \tag{28}$$

Combining (27) and (28), we get $q\lambda^h > \frac{1-q}{q}\lambda^h \Rightarrow q(1+q) > 1$, which contradicts our assumption about the value of q. It completes the proof.

□

Lemmas A4 and A5 imply that the a two-signal experiment with $\{h, l\}$ must be optimal. In this experiment, the investor completely disregards her own signal. It is easy to see that the optimal two-signal experiment has a threshold scheme, where the threshold $\bar{X}(q)$ should satisfy (4). The rest of the results for this part are similar to Lemma 1.

Proof of Part (b) Note that $\mathcal{I}(q) = \mathbb{I}_{\{X \ge \bar{X}(q)\}}$ and it is easy to see that $\bar{X}(q)$ is strictly increasing in q. Since $\bar{X}(q) < I$ for ever $q \in [\frac{1}{2}, \hat{q}]$, the result follows.

A6. Proof of Lemma 3

Proof for Part (a)

According to equation (5), the entrepreneur's utility from experiment $(\mathcal{Z}, \pi)$ is given by:

$$U^E(\mathcal{Z}, \pi) = \Sigma_{z\in\mathcal{Z}}\mathbb{E}[\varepsilon + (1-\mu)(X - I)|z]\mathbb{I}_{\{\mathbb{E}[X|z]\ge I\}} \tag{29}$$

Therefore, the entrepreneur does not induce investment when $X < I - \frac{\varepsilon}{1-\mu}$. If $\bar{X} \ge I - \frac{\varepsilon}{1-\mu}$, this constraint is not binding, so it does not change the equilibrium experimentation and payoffs. If $\bar{X} < I - \frac{\varepsilon}{1-\mu}$, then the entrepreneur optimally sends a high signal iff $X \ge I - \frac{\varepsilon}{1-\mu}$. Note that inducing investment is feasible since:

$$\int_{I-\frac{\varepsilon}{1-\mu}}^{1}(X-I)f(X)dx > \int_{\bar{X}}^{1}(X-I)f(X)dX = 0$$

Proof for Part (b) The interim rent of the insider is given by:

$$\bar{K}_S^{\varepsilon}(\mu) = \mu\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq\max\{\bar{X},I-\frac{\varepsilon}{1-\mu}\}\}}] \tag{30}$$

Therefore, the interim rent is increasing in $[1-\frac{\varepsilon}{I-\bar{X}},1]$. For $\mu<1-\frac{\varepsilon}{I-\bar{X}}$, the first and second derivatives with respect of μ are:

$$\begin{aligned}\bar{K}_S'(\mu) &= \int_{I-\frac{\varepsilon}{1-\mu}}^{1}(X-I)f(X)dX - \frac{\mu\varepsilon^2}{(1-\mu)^3}f(I-\frac{\varepsilon}{1-\mu})\\ \bar{K}_S''(\mu) &= \frac{\varepsilon^2}{(1-\mu)^5}[-(2+\mu)(1-\mu)f(I-\frac{\varepsilon}{1-\mu})+\mu\varepsilon f'(I-\frac{\varepsilon}{1-\mu})]\end{aligned} \tag{31}$$

Note that $\bar{K}_S'(0)>0$, $\bar{K}_S'(1-\frac{\varepsilon}{\bar{X}})<0$ and for small enough values of ε and continuous pdf functions, $\bar{K}_S''(\mu)<0$ over $\mu\in[0,1-\frac{\varepsilon}{I-\bar{X}}]$. Therefore, $\bar{K}_S(\mu)$ is non-monotone. Furthermore, it is concave for $\mu\in[0,1-\frac{\varepsilon}{I-\bar{X}}]$, for small enough values of ε. Since $\bar{K}_S(\mu)=0$ for $\mu\geq 1-\frac{\varepsilon}{I-\bar{X}}$, it implies it is unimodal for this range of ε. It completes the proof.

A7. Proof of Proposition 2

Proof for Parts (a) and (b)

The proof is similar to the proof for part (a) in Lemma 3.

Proof for Part (c) We can rewrite $\bar{K}_C^{\varepsilon}(\mu;q)=\mu\mathbb{E}[(\min\{X,D\}-I)\mathbb{I}_{\{X\geq\max\{\bar{X}(q),I-\frac{\varepsilon}{1-\mu}\}\}}]$, and define

$$\begin{aligned}\mu^l &= \sup\{\mu|\bar{K}_S^{\varepsilon}\text{ is increasing over }[0,\mu]\}\\ \mu^h &= \inf\{\mu|\bar{K}_S^{\varepsilon}\text{ is decreasing over }[\mu,1-\frac{\varepsilon}{I-\bar{X}}]\}\end{aligned} \tag{32}$$

Since $\bar{X}(q)<I-\varepsilon$, there exists $\hat{\mu}(q)$ such that $\bar{X}(q)=I-\frac{\varepsilon}{1-\hat{\mu}(q)}$. Because $\bar{X}(q)$ is strictly increasing in q, $\hat{\mu}(q)$ is strictly decreasing in q. If $\hat{\mu}(q)<\mu^l$, then $\bar{K}_C^{\varepsilon}(\mu;q)$ is increasing over $[0,1]$. If that is not the case, then the function is U-shape over $[\mu^h,1]$.

Proof for Part (d) Note that according to (31), as ε goes to zero, $\bar{K}_S^{\varepsilon}(\mu)$ becomes strictly increasing in $[0,1]$. Therefore, μ^l converges to one as ε goes to zero.

A8. Proof of Lemma 4

Proof for Part (a)

For the first part, see the proof of Lemma 1. For the second part, suppose the investor can contract on the realization of some interim event $a\in\mathcal{A}$. Then denote the conditional distribution of an event $a\in\mathcal{A}$ by $g(a|X)$, $X\in[0,1]$. Since we assume that without investor sophistication, the investor has no access to any interim signal that shows if the project is positive NPV, then have $\mathbb{E}[X-I|a]<0$, for every $a\in\mathcal{A}$.

Suppose the investor makes the continuation decision contingent on realization $a^+\in\mathcal{A}$, and the project has non-negative NPV. Now, we solve for the optimal experiment. The entrepreneur's expected payoff from signal $z \in \mathcal{Z}$ from the experiment $(\mathcal{Z}, \pi)$ is:

$$\begin{aligned} U^E(z; (\mathcal{Z}, \pi)) &= P(a^+|z)\mathbb{E}[\varepsilon + X - s(X)|a^+, z]\mathbb{I}_{\{\mathbb{E}[X-I|a^+,z]\geq 0\}} \\ = \frac{1}{Prob(z)} &\int_0^1 (\varepsilon + X - s(X))\pi(z|X)g(a^+|X)f(X)dX\mathbb{I}_{\{\mathbb{E}[X-I|a^+,z]\geq 0\}} \end{aligned} \tag{33}$$

By comparing (33) with (19), we see that setting milestone only changes the prior from $f(X)$ to $g(a^+|X)f(X)$. Therefore, the result of Proposition 1 follows, since $\mathbb{E}[X - I|a^+] < 0$.

Proof for Part (b)

For a given security, like $s(X)$, suppose for $\lambda = 1$, the entrepreneur uses the threshold $\bar{X}_s$ for sending high signals. Then, similar to the argument given in lemma 3, the insider receives positive interim payoff if:

$$\varepsilon + \bar{X}_s - (s_I(\bar{X}_s) + s_O(\bar{X}_s)) + (1 - \lambda)(s_O(\bar{X}_s) - I) < 0 \tag{34}$$

In short, condition (34) shows that the entrepreneur's gain from the continuation at $X = \bar{X}_s$ does not cover the marginal drop in the price of the security due to the inefficient continuation, $(1 - \lambda)(s_O(\bar{X}_s) - I)$. Therefore, the insider can get positive interim rent.

Proof for Part (c)

This follows directly from the necessary conditions given in Proposition 3 for implementing the first best information provision.

A9. Proof of Proposition 3

Note that the social surplus from the relationship financing is bounded by

$$U^E_{FB} = \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{\varepsilon + X - I \geq 0\}}] - K \tag{35}$$

Therefore for any security and any level of sophistication, the entrepreneur's surplus does not exceed this amount. As follows, we formalize this point and show that this surplus is achievable for the entrepreneur for any level of the investor's sophistication.

For every triplet $\{s_I(.), s_O(.), \lambda\}$ that satisfies Constraint (9), we have the following inequalities for any level of investor's sophistication:

$$\begin{aligned} U^E &= \mathbb{E}[M(X; s_I, s_O, \lambda)\mathcal{I}(X)] \leq \mathbb{E}[M(X; s_I, s_O, \lambda)\mathbb{I}_{\{M(X;s_I,s_O,\lambda)\geq 0\}}] \\ &= \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{M(X;s_I,s_O,\lambda)\geq 0\}}] - \mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{M(X;s_I,s_O,\lambda)\geq 0\}}] \\ &\leq \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{\varepsilon + X - I \geq 0\}}] - K = U^E_{FB} \end{aligned}$$

Therefore, the entrepreneur's expected payoff does not exceed U^E_{FB}, for all $q \in [\frac{1}{2}, 1]$. Now, we claim that a contract that satisfies the constraints in (9) is optimal if $s_I(I - \varepsilon) = \lambda I$. To see this, note that $M(X; s_I, s_O, \lambda)$ is strictly increasing in X. Therefore, we only need to show $M(I - \varepsilon; s_I, s_O, \lambda) = 0$, which is equivalent to $s_I(I - \varepsilon) = \lambda I$.

For $q = \frac{1}{2}$, the continuation channel never binds when the investor receives positive interim payoff, $s_I(I - \varepsilon) = \lambda I$ becomes a necessary condition, as well. Suppose $\varepsilon \in [0, \bar{\varepsilon}]$. Then, for all robust results designs, we need to have $s(I - \varepsilon) = \lambda I$, for all $\varepsilon \leq \bar{\varepsilon}$. Therefore, in all robust designs, the investor receives $s_I(X) = \lambda I$, for all $X \in [I - \bar{\varepsilon}, I]$.

Since the insider investor never makes any loss for the robust optimal securities and $q < 1$, they always receive positive expected payoff from continuation if they receive a high signal from the entrepreneur's experimentation. In other words, the investor disregards her own independent signal for the continuation decision.

A10. Proof of Corollary 2

The rent that the insider gets from the contract $\{s_I^{cb}(X), s_O^{cb}(X), \lambda\}$ is $\lambda\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq I\}}]$. Since $\lambda^{cb}(K)$ should be such that the insider receives expected payoff exactly K, then $\lambda^{cb}(K) = \frac{K}{\mathbb{E}[\max\{X-I,0\}]}$.

A11. Proof of Proposition 4

As it is shown for lemma 2 and 3, for $q \in [\frac{1}{2}, \hat{q}]$ and every long-term contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$, the entrepreneur optimally uses a threshold strategy for his experimentation. Moreover, the single-security constraint implies that the entrepreneur chooses from contracts in form of $\{\lambda s(X), (1-\lambda)s(X), \lambda\}$. Therefore, every contract can be represented by the pair of $\{s(X), \lambda\}$.

Denote the entrepreneur's indirect utility from investment at X by $M^s(s, \lambda)$, where:

$$M^s(X; s, \lambda) = \varepsilon + X - \lambda s(X) - (1-\lambda)I = M(X; \lambda s(.), (1-\lambda)s(.), \lambda) \tag{36}$$

Therefore, the entrepreneur solves the following maximization problem:

$$\begin{aligned} &\max_{s(.),\lambda,X_c} \mathbb{E}[M^s(X; s, \lambda)\mathbb{I}_{\{X\geq X_c\}}] \\ &s.t. \quad \lambda\mathbb{E}[(s(X)-I)\mathbb{I}_{\{X\geq X_c\}}] \geq K \\ &q\mathbb{E}[(s(X)-I)\mathbb{I}_{\{X\in[X_c,I)\}}] + (1-q)\mathbb{E}[(s(X)-I)\mathbb{I}_{\{X\in[I,q]\}}] \geq 0 \end{aligned} \tag{37}$$

Note that in (37), both the objective function and the first constraint are independent of q. Particularly, for small enough values of q, the first constraint implies the second constraint. In this case, "payoff channel" determines the optimal security. As q increases, the second constraint might bind. In this case, the "continuation channel" is at work.

Therefore, to prove the proposition, we first separately consider two cases that the second constraint does not and does bind, respectively. In other words, we find the optimal contract and the equilibrium payoffs that exploit the payoff channel and the continuation channel. Then, for each of these channels, we provide conditions on q that makes them preferred by the entrepreneur.

Payoff Channel

First we shut down the continuation channel to see the optimal contract that implements the payoff channel. Again we remind the readers that the outsiders pay competitive price $\mathbb{E}[s_O(X)]$, and the entrepreneur has to use $(1-\lambda)I$ of it for financing the investment and pays $s_O X$ when the project pays off, effectively netting the entrepreneur a cost of $(1-\lambda)I$.

Therefore, we are interested in the solution of the following maximization problem:

$$\max_{s(.),\lambda} \mathbb{E}[M^s(X; s, \lambda)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \quad s.t. \quad \lambda\mathbb{E}[(s(X)-I)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \geq K \tag{38}$$

Note that the objective function in (38) is bounded by $\mathbb{E}[\max\{\varepsilon+X-I, 0\}]$ and the constraint constitutes a closed and bounded subset in a $\mathcal{L}^1 \times [0,1]$ space that contains all combination of regular securities and λ. Therefore, the optimal contract exists. We denote the security $s(X) = X$, by $s_i(\cdot)$.

Definition 1. *Consider the contract $\{s(.), \lambda\}$. If $M^s(0; s, \lambda) < 0$, then $\hat{X}(s(.), \lambda)$ solves $M^s(\hat{X}(s(.), \lambda); s, \lambda) = 0$; otherwise, $\hat{X}(s(.), \lambda) = 0$.*

In fact, $\hat{X}(s(.), \lambda)$ is the threshold that the entrepreneur wants to use if the continuation channel is completely absent. Since $s(X) \leq X$ and $\varepsilon > 0$, $M^s(I-\varepsilon; s, \lambda) > 0$, then $\hat{X}(s(.), \lambda) < I - \varepsilon$, i.e. The first best cannot be implemented through the continuation channel under the single-security constraint.

Lemma (A6). *Suppose $s_1(.)$ and $s_2(.)$ are two regular securities such that $s_1(X) \geq s_2(X)\ \forall\ X \in [0,1]$.*
a) *$\hat{X}(s_1, \lambda)$ is weakly decreasing in λ.*
b) *$\hat{X}(s_1, \lambda) \geq \hat{X}(s_2, \lambda)$, for every $\lambda \in [0,1]$.*

Proof. It is clear from the expression (36) and definition 1. □

Now, we are ready to prove the optimality of equity for implementing the payoff channel. Consider a contract, like $\{s_1, \lambda_1\}$, that satisfies the constraint in (38). Part (b) in lemma A6 implies:

$$\lambda_1 \mathbb{E}[(s_1(X) - I)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\lambda_1)\}}] \leq \lambda_1 \mathbb{E}[(X - I)\mathbb{I}_{\{X \geq \hat{X}(s_i(.),\lambda_1)\}}] \tag{39}$$

Then there exists $\lambda_i \leq \lambda_1$ such that $\lambda_i \mathbb{E}[(X-I)\mathbb{I}_{\{X \geq \hat{X}(s_i(.),\lambda_i)\}}] = K$.
Now, we show that the entrepreneur prefers the contract $\{s_i(.), \lambda_i\}$ to $\{s_1, \lambda_1\}$:

$$\begin{aligned}
&\mathbb{E}[M(X; s_i, \lambda_i)\mathbb{I}_{\{X \geq \hat{X}(s_i(.),\lambda_i)\}}] = \mathbb{E}[(\varepsilon + (1-\lambda_i)(X-I))\mathbb{I}_{\{X \geq \hat{X}(s_i,\lambda_i)\}}] = \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(s_i,\lambda_i)\}}] - K \\
&\geq \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\mu_1)\}}] - \lambda_1 \mathbb{E}[(s_1(X) - I)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\lambda_1)\}}] = \mathbb{E}[M(X; s_1, \lambda_1)\mathbb{I}_{\{X \geq \hat{X}(s_1(.),\lambda_1)\}}]
\end{aligned}$$

Therefore, the optimal contract the implements the payoff channel is equity.

Continuation Channel

Now, we show that in the absence of the payoff channel, a call option is the optimal contract.

Definition 2. *For a contract $\{s(.), \lambda\}$, define $C(s)$ to be the solution to*

$$q \int_{C(s)}^{I} (s(X) - I) f(X) dX + (1-q) \int_{I}^{1} (s(X) - I) f(X) dX = 0, \tag{40}$$

if exists; otherwise, $C(s) = 0$.

In short, $C(s)$ is the threshold that the entrepreneur would use under the contract $\{s(.), \lambda\}$. In this part, we are interested to the solution of the following maximization problem:

$$\max_{s(.),\lambda} \mathbb{E}[M^s(X; s, \lambda)\mathbb{I}_{\{X \geq C(s)\}}] \quad s.t. \quad \lambda \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq C(s)\}}] \geq K \tag{41}$$

The following remarks are useful for our analysis.

Remark 1. *(40) implies the following relations:*

$$\int_{C(s)}^{I} (I - s(X)) f(X) dX = \frac{1-q}{2q-1} \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq C(s)\}}] \tag{42}$$

$$\int_{I}^{1} (s(X) - I) f(X) dX = \frac{q}{2q-1} \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq C(s)\}}] \tag{43}$$

Remark 2. *Equation (43) implies that the second constraint in (37) cannot be binding if the following inequality holds:*

$$\mathbb{E}[\max\{X - I, 0\}] < \frac{qK}{2q-1} \tag{44}$$

Therefore, throughout of this part we assume that q is big enough that the (44) does not hold.

Assumption 2. *ε is small enough that the following condition holds:*

$$F(I) - F(I-\varepsilon) < \frac{(1-q)K}{(2q-1)I}$$

Assumption 2 implies that for all regular securities $s(\cdot)$, $C(s) < I-\varepsilon$. We show that under this condition, a call option is uniquely optimal. However, we show that even if this this condition does not hold, a call option is still optimal, while it might not be unique.

Lemma (A7). *Under assumption 2, the optimal contract is the one that has the highest $C(s)$ and the condition in (41) is binding for $\{s, \lambda\}$.*

Proof. First, we show that under assumption 2, $C(s) < I - \varepsilon$. Equation (42) and the constraint in (41) imply:

$$\int_{C(s)}^{I} (I - s(X)) f(X) dX \geq \frac{(1-q)K}{(2q-1)\lambda}$$
$$\Rightarrow \int_{I-\varepsilon}^{I} (I - s(X)) f(X) dX \leq I(F(I) - F(I-\varepsilon)) < \frac{(1-q)K}{(2q-1)} \leq \frac{(1-q)K}{(2q-1)\lambda} = \int_{C(s)}^{I} (I - s(X)) f(X) dX$$
$$\Rightarrow C(s) < I - \varepsilon$$

Now, we only need to to show that the entrepreneur's expected payoff from a contract $\{s, \lambda\}$ is an increasing function of $C(s)$. We can assume the constraint in (41) is binding, otherwise, the entrepreneur can increase its expected payoff by decreasing λ. The entrepreneur's expected payoff is given by:

$$\begin{aligned}\mathbb{E}[M^s(X; s, \lambda)\mathbb{I}_{\{X \geq C(s)\}}] &= \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq C(s)\}}] - \lambda \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq C(s)\}}] \\ &= \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq C(s)\}}] - K\end{aligned}$$

Since $C(s) < I - \varepsilon$, it is clear that it is increasing in $C(s)$, provided the constraint is binding.

□

Now, We show that contract $\{s_N, 1\}$ is uniquely optimal under assumption 2, where $s_N = \max\{X - N, 0\}$ is a call option and N satisfies the following condition:

$$\frac{qK}{(2q-1)} = \mathbb{E}[(X - N - I)\mathbb{I}_{\{X - I \geq 0\}}], \tag{45}$$

First note that $X - s(X)$ is a weakly increasing function. Therefore,

$$\begin{aligned}&\int_{I}^{1} (I - s(I)) f(X) dX \leq \int_{I}^{1} (X - s(X)) f(X) dX \\ =&\int_{I}^{1} (X - I) f(X) dX - \int_{I}^{1} (s(X) - I) f(X) dX \\ \leq&\mathbb{E}[\max\{X - I, 0\}] - \frac{qK}{2q-1}\end{aligned} \tag{46}$$

where for the final inequality, we exploit the following inequality resulting from (43) and the condition in (41):

$$\int_{I}^{1} (s(X) - I) f(X) dX \geq \frac{qK}{(2q-1)\lambda} \geq \frac{qK}{(2q-1)}$$

Note that in (46), the equality holds only if $X - s(X) = I - s(I)$ for every $X \geq I$. Therefore, $X - s(X)$ should be constant if the inequality with equality holds. Moreover, (46) and (45) imply $I - s(I) \leq N$. It implies that $s(X) \geq X - N$ for every $X \leq I$ and every regular security that satisfies the constraint. In other words, $s(X) \geq s_N(X)$ for $X \leq I$. Moreover, it is easy to see that $s_N(X)$ binds the constraint. Now, we are ready to show that $C(s_N) > C(s)$ for every regular security. First, note that according to (42), for every security that binds the constraint in (41), we have:

$$\begin{aligned}\int_{C(s_N)}^{I} (I - s_N(X))f(X)dX = \frac{(1-q)K}{2q-1} = \int_{C(s)}^{I} (I - s(X))f(X)dX < \int_{C(s)}^{I} (I - s_N(X))f(X)dX \\ \Rightarrow C(s_N) > C(s)\end{aligned} \tag{47}$$

Therefore, according to lemma A7 and inequality (47), under assumption 2, a call option is uniquely optimal. Now, we show that even if condition (2) does not hold, a call option is still optimal, but potentially with a smaller λ.

Lemma (A8). *(i) For every regular security $s(.)$, there exists $\tilde{N}$ such that $C(s) = C(s_{\tilde{N}})$, where $s_{\tilde{N}}(X) = \max\{X - \tilde{N}, 0\}$. (ii) If $C(s) = C(s_{\tilde{N}})$, then $\mathbb{E}[(s_{\tilde{N}}(X) - s(X))\mathbb{I}_{\{X \geq C(s)\}}] \geq 0$.*

Proof. It is easy to see that $C(s) \geq C(s_0)$ and $C(s_{\tilde{N}}) \to I$ as $\tilde{N}$ goes to 1. Moreover, by definition $C(s) \leq I$ for every regular security. Therefore, the continuity of $C(s_{\tilde{N}})$ in $\tilde{N}$ proves the part (i). The idea for part (ii) is similar to the inequality (47). □

Therefore, for the optimal contract $\{s^*, \lambda^*\}$, there is another contract, like $\{s_{\tilde{N}}, \lambda_{\tilde{N}}\}$, such that $s_{\tilde{N}}(\cdot)$ is a call option, $\lambda_{\tilde{N}} \leq \lambda^*$ (according to part (ii) in lemma A8) and it implements the same expected payoffs for the entrepreneur and the investors. Therefore, call options optimally implement the continuation channel. For the rest of proof, we denote this optimal call option as $\{s_N, \lambda_N\}$.

Equity (Payoff Channel) vs Call Option (Continuation Channel)

To find the optimal security, we only need to compare the entrepreneur's expected payoff from the equity contract $\{s_i, \lambda_i\}$ and the call option contract $\{s_N, \lambda_N\}$. In other words, the entrepreneur chooses the former iff (48) holds:

$$\begin{aligned}\mathbb{E}[M^s(X; s_i, \lambda_i)\mathbb{I}_{\{X \geq \hat{X}(s_i,\lambda_i)\}}] \geq \mathbb{E}[M^s(X; s_N, \lambda_N)\mathbb{I}_{\{X \geq C(s_N)\}}] \\ \Rightarrow \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(s_i,\lambda_i)\}}] - K \geq \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X \geq C(s_N)\}}] - K\end{aligned} \tag{48}$$

Suppose $\underline{q}$ is such that:

$$\mathbb{E}[\max\{X - I, 0\}] = \frac{\underline{q}K}{2\underline{q} - 1}$$

In this case $N = 0$, therefore $C(s_N) = \bar{X}$. If there exists no $q \in [\underline{q}, \hat{q}]$ for which $C(s_N) = \hat{X}(s_i, \lambda_i)$, continuity of $C(s_N)$ in q implies that the second constraint in (37) never binds. Therefore, the proposition holds for $q^s = \hat{q}$. Otherwise, denote q^s is the value at which $C(s_N) = \hat{X}(s_i, \lambda_i)$. Clearly the equity is optimal for $q \leq q^s$. For $q > q^s$ that satisfy the condition (2), the argument is straightforward, sine $C(s_N)$ is weakly increasing in q, while $\hat{X}(s_i, \lambda_i)$ is independent of q. If condition (2) does not hold and $C(s_N) = I - \varepsilon$, optimality of the call option is clear. If $C(s_N) > I - \varepsilon$, then $C(s) > I - \varepsilon$ for every regular security according to lemma A8. In this case, the payoff channel is never binding, since as it is shown earlier $\hat{X}(s, \lambda) < I - \varepsilon$ for any contract. It completes the proof.

A12. Proof of Corollary 3

We are searching over contracts with λ greater than some $\underline{\lambda} \in (0,1)$. We consider the case that $\underline{\lambda} > \lambda_i$, as otherwise the outsiders' financial constraint would not be binding. We show that among theses contracts, a debt contract optimally implements the payoff channel. Similar to the argument in the proof of proposition 4, for small enough values of q, the entrepreneur prefers to implement the payoff channel.

Suppose debt contract $d_{D(\underline{\lambda})}(X) = \min\{X - D(\underline{\lambda}), 0\}$ is such that $\underline{\lambda}\mathbb{E}[(d_{D(\underline{\lambda})}(X) - I)\mathbb{I}_{\{X \geq \hat{X}(d_{D(\underline{\lambda})}, \underline{\lambda})\}}] = K$.

According to Definition 1, $\hat{X}(d_{D(\underline{\lambda})}, \underline{\lambda}) = \hat{X}(s_i, \underline{\lambda})$, and by a similar argument as we have in the proof of proposition 4, $\hat{X}(s_i, \underline{\lambda}) \geq \hat{X}(s, \lambda)$, for any contract $\{s(.), \lambda\}$, where $\lambda \geq \underline{\lambda}$. Therefore the entrepreneur weakly prefers $\{d_{D(\underline{\lambda})}, \lambda\}$, since:

$$\begin{aligned}
&\mathbb{E}[M^s(X; d_{D(\underline{\lambda})}, \underline{\lambda})\mathbb{I}_{\{X \geq \hat{X}(d_{D(\underline{\lambda})}, \underline{\lambda})\}}] = \mathbb{E}[(X - I + \varepsilon)\mathbb{I}_{\{X \geq \hat{X}(d_{D(\underline{\lambda})}, \underline{\lambda})\}}] - K \\
&\geq \mathbb{E}[(X - I + \varepsilon)\mathbb{I}_{\{X \geq \hat{X}(s,\lambda)\}}] - \lambda\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \hat{X}(s,\lambda)\}}] = \mathbb{E}[M^s(X; s, \lambda)\mathbb{I}_{\{X \geq \hat{X}(s,\lambda)\}}]
\end{aligned}$$

A13. Proof of Proposition 5

Proof for Part (a)

Note that the investor, after observing signal z, solves the following maximization problem:

$$\max_{\alpha \in [0,1]} \mathbb{E}[s(\alpha, r(\alpha)X)|z] - \alpha I \tag{49}$$

We also know

$$\mathbb{E}[s(\alpha, r(\alpha)X)|z] - \alpha I \leq r(\alpha)\mathbb{E}[s(1, X)|z] - \alpha I \leq \alpha(\mathbb{E}[s(1, X)|z] - I) \leq \max\{\mathbb{E}[s(1, X)|z] - I, 0\}$$

Hence it is optimal to choose $\alpha \in [0, 1]$. The rest of proof is straightforward following Lemma 1.

Proof for Part (b)

Suppose $\alpha^*(z)$ is the investor's level of investment scale given the signal z. If $\alpha^*(z) > 0$ for some z, then the FOC implies $r'(\alpha^*(z))\mathbb{E}[s(X)|z] \geq I$.

Then $U^I(z; (\mathcal{Z}, \pi)) = r(\alpha^*(z))\mathbb{E}[s(X)|z] - \alpha^*(z)I \geq (r(\alpha^*(z)) - \alpha^*(z)r'(\alpha^*(z)))\mathbb{E}[s(X)|z]$.

Note that $r(.)$ is concave in $[0, \alpha^*(z)]$, therefore $r(\alpha^*(z)) - \alpha^*(z)r'(\alpha^*(z)) > r(0) = 0$.

Clearly $\mathbb{E}[s(X)|z] > 0$. Therefore, the investor receives positive interim payoff from a signal z if $r'(0)\mathbb{E}[s(X)|z] > I$. Note, that the entrepreneur strictly prefers investment to not investment. Therefore, he can introduce a signal that induces investment with positive probability iff $r'(0)s(1) > I$. In this case, the investor receives positive rent.

A14. Proof of Proposition 6

Proof for Part (a)

For the projects with full benefit of scale, the argument follows the proof of lemma 4(a), because the investor optimally follows a binary strategy. Therefore, we only need to show the result for the DRS case.

Denote α^* as one of the socially optimal actions, which solves the following maximization problem:

$$\alpha^* \in \operatorname*{argmax}_{\alpha} \mathbb{E}[(r(\alpha)(X + \varepsilon) - \alpha I)\mathbb{I}_{\{X \geq \frac{\alpha}{r(\alpha)}I - \varepsilon\}}]$$

Moreover, define $\tilde{S}(\alpha, \hat{X})$, $\tilde{V}^I(\alpha, \hat{X})$ and $\tilde{V}^E(\alpha, \hat{X})$ as the expected welfare, the investor's expected payoff, and the entrepreneur's expected payoff respectively, given that the entrepreneur sends a high signal for $X \geq \hat{X}$ and the investor chooses α.

If the security implements the first-best scale, then $(\alpha^*, \frac{\alpha^*}{r(\alpha^*)}I - \varepsilon)$ maximizes both $\tilde{S}(\alpha, \frac{\alpha^*}{r(\alpha^*)}I - \varepsilon)$ and $\tilde{V}^I(\alpha, \frac{\alpha^*}{r(\alpha^*)}I - \varepsilon)$. Now, we apply the envelope theorem on $\hat{X}$ at $\hat{X} = \frac{\alpha^*}{r(\alpha^*)}I - \varepsilon$ and show that $\frac{\partial \tilde{V}^E(\alpha,\hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} < 0$, therefore the entrepreneur would be better off by decreasing the threshold.

$$\frac{\partial \tilde{S}(\alpha^S(\hat{X}), \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} = \frac{\partial \tilde{S}(\alpha^*, \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} = 0 \geq \frac{\partial \tilde{S}(\alpha^I(\hat{X}), \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} \tag{50}$$

$$\begin{aligned} \frac{\partial \tilde{V}^I(\alpha^I(\hat{X}), \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} &= \frac{\partial \tilde{V}^I(\alpha^*, \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} \\ &= -(s(\alpha^*, r(\alpha^*)\left(\frac{\alpha^*}{r(\alpha^*)}I - \varepsilon)) - \alpha^* I\right) f\left(\frac{\alpha^*}{r(\alpha^*)}I - \varepsilon\right) > 0, \end{aligned} \tag{51}$$

where $\alpha^S(\hat{X})$ and $\alpha^I(\hat{X})$ denote the socially optimal and the investor's optimal scaling decision, respectively. The inequality (50) follows from the Envelope theorem and the optimality of $(\alpha^*, \frac{\alpha^*}{r(\alpha^*)}I - \varepsilon)$. The final inequality in (51) results from $s(\alpha, r(\alpha)X) \leq r(\alpha)X$ for every $\alpha, X \in [0,1]$. By combining (50) and (51),

$$\frac{\partial \tilde{V}^E(\alpha^I(\hat{X}), \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} = \frac{\partial \tilde{S}(\alpha^I(\hat{X}), \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} - \frac{\partial \tilde{V}^I(\alpha^I(\hat{X}), \hat{X})}{\partial \hat{X}}|_{\hat{X}=\frac{\alpha^*}{r(\alpha^*)}I-\varepsilon} < 0. \tag{52}$$

Consequently, the pair $(\alpha^*, \frac{\alpha^*}{r(\alpha^*)}I - \varepsilon)$ cannot constitute an equilibrium, because the entrepreneur finds it beneficial to decrease the threshold. It contradicts the earlier assumption that the security implements the first-best outcome.

Proof for Part (b)

The argument for the projects with full benefit of scale follows from Proposition 3, because the insider optimally follows a binary investment strategy. For DRS projects, define $\alpha^*(\varepsilon)$ as the highest socially optimal level of scale, namely the largest solution to the following maximization problem:

$$\max_{\alpha} \mathbb{E}[\max\{r(\alpha)(X + \varepsilon) - \alpha I, 0\}]$$

Now, consider a regular security $s(\alpha, r(\alpha))$:

$$s(\alpha, r(\alpha)X) \geq \alpha I \quad \forall \alpha, X \in [0,1], \quad \lambda(\alpha)\mathbb{E}[s(\alpha, r(\alpha)X) - \alpha I] = K, \quad s_I(\alpha, r(\alpha)X) = \lambda(\alpha)s(\alpha, r(\alpha)X).$$

Note that $s(\cdot, \cdot)$ is a convertible security for every values of $\alpha \in [0,1]$. Moreover, the share of the insider from the investment depends on her scaling decision through $\lambda(\alpha)$, so she is indifferent between all scaling decisions. Therefore, choosing the optimal scaling is a subgame equilibrium. Because the entrepreneur's expected payoff is the total expected payoff minus K, he chooses the socially optimal experiment, i.e. sending a high signal for $X \geq \frac{\alpha^*(\varepsilon)}{r(\alpha^*(\varepsilon))}I - \varepsilon$.

A15. Proof of Lemma 5

First, we show that the two-signal experiment introduced in Lemma 2(a) is not optimal for big enough values of q. Then, we show that when a three-signal experiment is optimal, a nested interval structure is used for providing endogenous information.

Non-optimality of two-signal experiments

Suppose the contrary holds that a threshold scheme, with threshold $\bar{X}(q)$ (as introduced in (4)) is optimal. Then Lemma A1 says it is the optimal experiment among all two-signal experiments that the high signal always induce investment. Now consider η_1, η_2 and η_3 that satisfy the following conditions:

$$q\int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2}(s(X)-I)f(X)dX+(1-q)\int_{1-\eta_3}^{1}(s(X)-I)f(X)dX\leq 0$$

$$(1-q)\int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2}(s(X)-I)f(X)dX+q\int_{1-\eta_3}^{1}(s(X)-I)f(X)dX\geq 0$$

If the two-signal experiment is optimal, then the following three-signal experiment should implement a suboptimal investment function for the entrepreneur.

$$\tilde{\pi}(h|X)=\begin{cases}1 & X\in[\bar{X}(q)+\eta_2,1-\eta_3)\\ 0 & X\in[0,\bar{X}(q)+\eta_2)\cup[I-\eta_3,1]\end{cases}\qquad \tilde{\pi}(m|X)=\begin{cases}1 & X\in[\bar{X}(q)-\eta_1,\bar{X}(q)+\eta_2)\cup[1-\eta_3,1]\\ 0 & X\in[0,\bar{X}(q)-\eta_1)\cup[\bar{X}(q)+\eta_2,1-\eta_3)\end{cases}$$

Therefore, $\eta_1=\eta_2=\eta_3=0$ should be the solution to the following optimization problem:

$$\max_{\eta_1,\eta_2,\eta_3}(1-q)\int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)}(\varepsilon+X-s(X))f(X)dX-(1-q)\int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2}(\varepsilon+X-s(X))f(X)dX-q\int_{1-\eta_3}^{1}(\varepsilon+X-s(X))f(X)dX$$

$$\begin{aligned}s.t.\quad q\int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2}(s(X)-I)f(X)dX+(1-q)\int_{1-\eta_3}^{1}(s(X)-I)f(X)dX\leq 0\\ (1-q)\int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2}(s(X)-I)f(X)dX+q\int_{1-\eta_3}^{1}(s(X)-I)f(X)dX\geq 0\end{aligned}$$

Suppose κ_1 and κ_2 are the Lagrange multipliers for the above constraints. The FOCs at $\eta_1=\eta_2=\eta_3=0$ are as follows:

$$\begin{aligned}[\eta_1]|_{\eta_1=0}=0\Rightarrow & f(\bar{X}(q))[(1-q)(\varepsilon+\bar{X}(q)-s(\bar{X}(q))+\kappa_2(s(\bar{X}(q))-I))]=0\\ \Rightarrow & \kappa_2=\frac{\varepsilon+\bar{X}(q)-s(\bar{X}(q))}{I-s(\bar{X}(q))}\end{aligned}\tag{53}$$

$$\begin{aligned}[\eta_2]|_{\eta_2=0}=0\Rightarrow & f(\bar{X}(q))[-q(\varepsilon+\bar{X}(q)-s(\bar{X}(q)))+q\kappa_1(s(\bar{X}(q))-I)+(1-q)\kappa_2(s(\bar{X}(q))-I)]=0\\ \Rightarrow & -\kappa_1=\frac{2q-1}{q}\frac{\varepsilon+\bar{X}(q)-s(\bar{X}(q))}{I-s(\bar{X}(q))}=\frac{2q-1}{q}\kappa_2\end{aligned}\tag{54}$$

$$\begin{aligned}[\eta_3]|_{\eta_3=0}=0\Rightarrow & f(1)[-(1-q)(\varepsilon+1-s(1))+(\kappa_1(1-q)+\kappa_2 q)(s(1)-I)]=0\\ \Rightarrow & \frac{\varepsilon+1-s(1)}{s(1)-I}=\frac{\kappa_1(1-q)+\kappa_2 q}{1-q}=\frac{3q^2-3q+1}{q(1-q)}\frac{\varepsilon+\bar{X}(q)-s(\bar{X}(q))}{I-s(\bar{X}(q))}\end{aligned}\tag{55}$$

Note that:

$$\frac{\varepsilon+\bar{X}(q)-s(\bar{X}(q))}{I-s(\bar{X}(q))}\geq\frac{\varepsilon}{I}$$

Therefore, (55) implies that for every $q \in [\frac{1}{2}, 1]$, the following inequality holds:

$$\frac{I}{\varepsilon}\frac{\varepsilon + 1 - s(1)}{s(1) - I} \geq \frac{3q^2 - 3q + 1}{q(1-q)} \tag{56}$$

The RHS in (56) goes to infinity as $q \to 1$, while the LHS is constant. It is the contradiction with earlier assumption that the two-signal experiment is optimal. Therefore, a three-signal experiment is optimal for large enough values of q.

Nested Interval Structure

Guo and Shmaya (2017) prove the second part of the lemma in their Theorem 3.1 and Discussion 6.3, for securities, like $s(X)$, that $\frac{s(X)-I}{\varepsilon+X-s(X)}$ is increasing in X. Note that this condition holds for $s_i(X) = X$.

A16. Proof of Proposition 7

Proof for Part (a)

Suppose $\mu_2 > \mu_1$ and the equilibrium investment functions are respectively $\mathcal{I}_2(X)$ and $\mathcal{I}_1(X)$. We show that:

$$\mathbb{E}[(\varepsilon + X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + X - I)\mathcal{I}_2(X)] \tag{57}$$

Suppose the contrary. Note that both $\mathcal{I}_1(X)$ and $\mathcal{I}_2(X)$ are implementable for the entrepreneur (There are experiments that implement these investment functions), since q is fixed. The optimality of the investment functions implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] \tag{58}$$

Therefore, if (57) does not hold, (58) implies:

$$\mathbb{E}[(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(X - I)\mathcal{I}_1(X)] \tag{59}$$

Furthermore, the the fact that the investor receives positive interim payoff for $q > \frac{1}{2}$ and optimality of $\mathcal{I}_2(X)$ for μ_2 imply:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \tag{60}$$

Therefore, (58) and (60) result in:

$$\begin{aligned}
&\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \\
\geq&\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \\
\Rightarrow&\mathbb{E}[(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(X - I)\mathcal{I}_2(X)]
\end{aligned} \tag{61}$$

which contradicts (59). Therefore, (57) holds. It shows the equilibrium investment function become more socially efficient as μ decreases.

Proof for Part (b) As it is shown in lemma 5, there exists q^* such that for $q > q^*$, a three-signal experiment is optimal. Fix a level of sophistication in the range, like q. Therefore, there are functions $M_l(\mu)$, $H_l(\mu)$ and $H_h(\mu)$ such that the entrepreneur sends a high signal for $X \in [H_l(\mu), H_h(\mu)]$ and a medium signal in $[M_l(\mu), H_l(\mu)] \cup [H_h(\mu), 1]$. Moreover, with a similar argument as given in the proof of lemma 3(a), interim competition does not affect the entrepreneur's experimentation if $\varepsilon + (1-\mu)(M_l(1) - I) \geq 0$. Therefore, without loss of generality, we assume μ is small enough that $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Moreover, as discussed earlier, the insider does not get any interim rent conditional on realization of a medium signal. Therefore, the insider's interim rent is as follows:

$$K_C(\mu;q) = \mu \int_{H_l(\mu)}^{H_h(\mu)} (X-I)f(X)dX = \frac{(2q-1)\mu}{q} \int_{I}^{H_h(\mu)} (X-I)f(X)dX \tag{62}$$

where the last equality comes from the fact the constraint for sending a high signal is binding. The following lemma characterizes the derivative with respect to μ when $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Lemma (A9). *If $\varepsilon + (1-\mu)(M_l(1) - I) < 0$, then*

$$\frac{\partial}{\partial \mu} K_C(\mu;q) = \frac{2q-1}{q} \left[\int_I^{H_h} (X-I)f(X)dX - \frac{q(1-q)}{2q-1} \frac{\mu\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) \right] \tag{63}$$

Proof. By taking derivative from (62) with respect to μ, we get:

$$\frac{\partial}{\partial \mu} K_C(\mu;q) = \frac{(2q-1)}{q} \int_I^{H_h(\mu)} (X-I)f(X)dX + H_h'(\mu) \frac{(2q-1)\mu}{q} (H_h(\mu) - I) f(H_h(\mu)) \tag{64}$$

We know that the conditions for sending high and medium signal binds for all values of μ. Therefore:

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ (1-q) \int_{I - \frac{\varepsilon}{1-\mu}}^{H_l} (X-I)f(X)dX + q \int_{H_h}^{1} (X-I)f(X)dX \right\} = 0 \\ &\Rightarrow - \frac{(1-q)\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) + (1-q)H_l'(H_l - I)f(H_l) - qH_h'(H_h - I)f(H_h) = 0 \end{aligned} \tag{65}$$

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ q \int_{H_l}^{I} (X-I)f(X)dX + (1-q) \int_{I}^{H_h} (X-I)f(X)dX \right\} = 0 \\ &\Rightarrow (1-q)H_h'(H_h - I)f(H_h) + qH_l'(H_l - I)f(H_l) = 0 \end{aligned} \tag{66}$$

By combining (65) and (66), we get:

$$H_h'(H_h - I)f(H_h) = -\frac{q(1-q)}{2q-1} \frac{\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu})$$

We get (63) by substituting the last equality in (64). □

Note that H_h and H_l are the solution to the following maximization problem, where $M_l = I - \frac{\varepsilon}{1-\mu}$:

$$\begin{aligned} \max_{H_l, H_h} \; & (1-q) \int_{M_l}^{H_l} (\varepsilon + (1-\mu)(X-I))f(X)dX + \int_{H_l}^{H_h} (\varepsilon + (1-\mu)(X-I))f(X)dX + q \int_{H_h}^{1} (\varepsilon + (1-\mu)(X-I))f(X)dX \\ s.t. \quad & (1-q) \int_{M_l}^{H_l} (X-I)f(X)dX + q \int_{H_h}^{1} (X-I)f(X)dX \geq 0 \\ & q \int_{H_l}^{I} (X-I)f(X)dX + (1-q) \int_{I}^{H_h} (X-I)f(X)dX \geq 0 \end{aligned}$$

According to lemma 5, we know that both constraints bind. It implies $H_l \to I$ and $H_h \to 1$, as $q \to 1$. Since $f(.)$ and $\frac{\mu}{(1-\mu)^3}$ are bounded ($\mu < 1 - \frac{\varepsilon}{I}$), then the result follows from Lemma A9.

A17. Proof of Proposition 9

We prove the case of single-security design without investor sophistication. The proof for the other case is similar.

Let $\lambda^I \in argmax_\lambda \;\; \lambda\mathbb{E}\left[(X-I)\mathbb{I}_{\left\{X \geq I - \frac{\varepsilon}{1-\lambda}\right\}}\right]$, then according to Corollary A5, the insider investor solves the following optimization problem:

$$\max_{s(.),\lambda} \lambda\mathbb{E}[(s(X)-I)\mathbb{I}_{\left\{X \geq \hat{X}(s,\lambda)\right\}}]$$

According to lemma A6, $\hat{X}(s,\lambda) \leq \hat{X}(s_i,\lambda) \leq I-\varepsilon$, where $\{s_i,\lambda_i\}$ is the optimal contract for the entrepreneur. Therefore, the investor uses security $s_i(X)$ in her contract. Therefore, the solution to the insider's optimization problem is $\{s_i(.),\lambda^I\}$. By definition, $\lambda^I > \lambda_i$. Therefore, $\hat{X}(s_i,\lambda^I) \leq \hat{X}(s_i,\lambda_i)$, which shows the investor's solution is less optimal than the entrepreneur's solution.

A18. Proof of Proposition 8

Proof for Part (a)

Secretly Manipulation of the Signal: First suppose the entrepreneur can secretly change the signal realization with probability $\alpha > 0$. Similar to Lemma 1, the entrepreneur follows a threshold strategy, i.e. There exists $\bar{X}_\alpha \in [0,1]$ such that the experiment generates a high signal for $X \geq \bar{X}_\alpha$, if he can implement investment with positive probability. Therefore, the the high signal induces investment if the investor receives non-negative expected payoff from the investment following the high signal, which is equivalent to (67).

$$\alpha \int_0^{\bar{X}_\alpha} (X-I)f(X)dX + \int_{\bar{X}_\alpha}^1 (X-I)f(X)dX \geq 0 \tag{67}$$

The first term in (67) shows the probability that the experiment generates a low signal, but the entrepreneur sends a high signal. Note that for $\alpha = 1$, the inequality does not hold for any threshold value. Therefore, there exists $\bar{\alpha} \in [0,1]$ above which the investment is not feasible. However, for $\alpha \leq \bar{\alpha}$, the entrepreneur chooses $\bar{X}_\alpha$ such that the inequality (67) binds, which implies the investor becomes indifferent between investment and not investment after receiving the high signal. Consequently, the investor receives zero interim rent for all values of $\alpha \in [0,1]$.

Random Monitoring: Now suppose the investor monitors the signal realization with probability $\beta < 1$. First, suppose the investor decides only based on her observed signal when she monitors, or in other words, the investor cannot commit to punish the entrepreneur for misreporting. In this case, there is no signal, like h, that always induces investment, because otherwise the entrepreneur would optimally always report h, which leads to negative expected payoff for the investor when she does not monitor. As such, the investor only invests when she monitors, which makes the entrepreneur's reporting strategy irrelevant. Therefore, for the experimentation, the entrepreneur would follow the threshold strategy at $\bar{X}$, in which the investor does not get interim rent.

Now consider the case that the investor commits to punish misreporting. in this case, the entrepreneur might use 4 kinds of signals: 1. low signals, like l_0, that never induce investment. 2. Low signals, like l_1, that only induce investment when they are no monitored. 3. high signals, like h_0, that only induce investment if they are verified. 4. High signals, like h_1, that always induce investment.

If $\beta < \frac{1}{2}$, then type h_0 has the incentive to misreport. It means that the investor makes a loss by investing without monitoring. So, she would be better off by not investing at all when she does not monitor. For $\beta \geq \frac{1}{2}$, note that the investor invests with monitoring when $h_0 \cup h_1$ realizes and invests without monitoring $h_1 \cup l_0 \cup l_1$

realizes. If the investor receives positive expected payoff from the event $l_0 \cup l_1 \cup h_1$ then the entrepreneur can expand h_1 without changing the reporting strategies of the types h_0, l_0 and l_1, and eliminating all the investor's interim rent. In equilibrium, the investor becomes indifferent between investment and not investment when she does not monitor. Similarly, it is easy to see that the investor cannot extract interim rent from $h_0 \cup h_1$, as they can be expanded in a way that eliminates all the investor's interim rent. (Note that as $\beta > \frac{1}{2}$, the entrepreneur prefers induce investment with monitoring rather than without monitoring, if both of them are not possible). Once again, the investor cannot extract any interim rent via random monitoring.

In particular, the equilibrium experiment has two thresholds $\hat{X}_{\beta 1}$ and $\hat{X}_{\beta 2}$, where signal l_1 is generated for $X \in [0, \hat{X}_{\beta 1}]$, signal h_0 is generated for $X \in (\hat{X}_{\beta 1}, \hat{X}_{\beta 2}]$ and the rest values of X generate h_1.

Proof for Part (b)

Secretly Manipulation of the Signal: Consider the contractual solutions introduced in Proposition 3. While they can align the entrepreneur's preferences with the social planner's in designing the experiment ex ante, they cannot ex post, specially when a low signal realizes. In particular, in the baseline case, the entrepreneur receives $\mathbb{E}[M(X; s_I, s_O, \lambda)|X \geq I - \varepsilon] > 0$ from the investment. It means if a low signal realizes, he would change the signal realization if he could. Therefore, no contractual solution can implement the first-best outcome, as they do not incentivize the entrepreneur to report truthfully. It implies for $\alpha > \bar{\alpha}$, no security has positive expected payoff in equilibrium, conditional on any signal realization. Consequently, relationship financing is infeasible for large enough values of α.

Now we solve for the optimal contract. If the project is invested with positive probability, then the expected payoff of the entrepreneur from the contract $\{s_I(.), s_O(.), \lambda\}$ is as follows:

$$U^E_\alpha(s_I(\cdot), s_O(\cdot), \lambda) = \mathbb{E}[M(X; s_I, s_O, \lambda)(\alpha + (1 - \alpha)\mathcal{I}(X))],$$

where $\mathcal{I}(\cdot)$ is the investment function for the case that the entrepreneur cannot secretly manipulate the signal. If $\mathbb{E}[\max\{X - I + \varepsilon, 0\}] > K$, then with an argument similar to the proof of proposition 3, the following set of convertible securities are optimal and robust to ε, for small enough values of α:

$$\lambda I \leq I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \ \forall\, X < I, \quad \mathbb{E}[(s_I(X) - \lambda I)(\alpha + (1 - \alpha)\mathbb{I}_{\{X \geq I - \varepsilon\}})] = K,$$
$$\mathbb{E}[(s_O(X) - (1 - \lambda)I)(\alpha + (1 - \alpha)\mathbb{I}_{\{X \geq I - \varepsilon\}})] = K, \quad 0 \leq s_O(X) \leq X - s_I(X) \ \forall\, X \in [0, 1]$$

Note that the design might not be robust to q, as the insider's payoff becomes sensitive to the downside realization of the final cashflow. However, it is still robust to the value of ε, as the insider's ex-post payoff does not depend on ε.

Random Monitoring: First suppose the investor cannot credibly threat the entrepreneur to terminate the project when he misreports. As it is discussed earlier, in equilibrium, the entrepreneur always report the high signal and the investor invests if an only if she verifies the signal is truly high. Therefore, the following convertible securities are optimal and robust to the values of ε and q.

$$\lambda I \leq I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \ \forall\, X < I, \quad \beta\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X \geq I - \varepsilon\}}] = K,$$
$$\mathbb{E}[(s_O(X) - (1 - \lambda)I)(\alpha + (1 - \alpha)\mathbb{I}_{\{X \geq I - \varepsilon\}})] = K, \quad 0 \leq s_O(X) \leq X - s_I(X) \ \forall\, X \in [0, 1]$$

Clearly, these securities are implementable for large enough values of β. For the case of credible punishments, a robust design might not exist, as the optimal experimentation involve three signals for large values of ε, while it involves two signals for the smaller values. However, for smaller values of ε, the convertible securities specified above are optimal and the equilibrium outcomes are similar.